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PERFORMANCE EVALUATION OF AN INFERENTIAL PREDICTIVE CONTROL ALGORITHM

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ABSTRACT

Within the chemical and process industries it is common to find plants suffering from the infrequent measurement of key process outputs due to sampling limitations. This contribution describes an 'integrated' inferential predictive controller that can overcome measurement limitations while offering the benefits of predictive control. The results of a rigorous performance evaluation of the inferential predictive controller are described when applied to a non-linear simulation of a distillation column. A direct application of the inferential controller is considered against a cascaded implementation. The inferential controller is tuned to meet robust performance objectives. The results of the simulation studies are used to develop heuristics for the implementation of the inferential controller. It is found that no extra benefits are obtained through the use of a cascade implementation.

INTRODUCTION

The inability to measure key process variables at a rate suitable for on-line control is a problem common to many industrial processes. The instrumentation may not exist hence requiring off-line laboratory assay or analysers with long cycle times have to be employed. The penalty is that deviations from nominal operation may remain undetected for significant periods of time. Effort towards alleviating this problem has included the development of inferential estimators [1,2] with considerable success when applied to industrial situations [3,4]. Inferential estimators are reliant on the *primary* variable being related to other more easily measured *secondary* variables. The secondary outputs are used to *infer* a value of the primary variable at the more frequent sampling rate of the secondary variables.

A natural progression of the work on inferential estimation is the development of an inferential controller. As inferential estimation is more justified in high performance applications, it follows that the corresponding use of modern predictive control techniques, such as Dynamic Matrix Control (DMC) [7] and Generalised Predictive Control (GPC) [8], is more appropriate than conventional feedback control. It has previously been shown [5] how the inferential estimator, introduced by Guilandoust et al [1], can be synthesised into the Generalised Minimum Variance (GMV) control algorithm [6]. Initial results were encouraging, and the inferential GMV controller was shown to significantly out-perform a 'standard' GMV controller.

This paper extends the work of Brunet-Manquat et al [5] and develops a long range predictive controller with inferential estimation capabilities. Robust control concepts are used to select controller parameters. A non-linear simulation of a binary distillation column is used to evaluate the performance capabilities of the inferential predictive controller; when it is implemented 'directly' and when it is implemented in a cascade strategy.

INFERENTIAL PREDICTIVE CONTROL

Inferential Estimation

Following Guilandoust et al [1], the inferential estimator aims to provide an estimate of, $y(t) = y_O(t-d)$, where $y(t)$ is the measured value of the primary output at time *t*, $y_O(t)$ is the *actual* value of the primary output at time *t*, and *d* is the measurement delay as an integer multiple of the secondary sample time. The primary and secondary models are:

$$\begin{aligned}\hat{y}(t+d) &= \beta_1 u(t-m-1)+\ldots+\beta_n u(t-m-n+1)+\\ &\alpha_0\hat{v}(t|t)+\ldots+\alpha_{n-1}\hat{v}(t-n+1|t-n+1)+\delta_1\varepsilon(t)+\ldots\\ &+\delta_{n-1}\varepsilon(t-n-2)\end{aligned} \tag{1}$$

$$\begin{aligned}\hat{v}(t) &= -a_1\hat{v}(t-1)-\ldots-a_n\hat{v}(t-n)+b_1u(t-m-1)+\ldots\\ &+b_nu(t-m-n)+k_1\varepsilon(t)+\ldots+k_n\varepsilon(t-n+1)\end{aligned} \tag{2}$$

where $\varepsilon(t) = v(t) - \hat{v}(t)$, $u(t)$ is the manipulated input, $\hat{y}(t+d)$ and $\hat{v}(t)$ are the respective estimates of the primary and secondary variable, '*m*' is the smallest time delay in the state response to changes in *u(t)*. The model coefficients can be identified using any suitable parameter estimation technique such as least-squares.

Model for Inferential Predictive Controller Design

Equations (1) and (2) may be rewritten as:

$$\hat{y}(t+d) = \beta u(t-1) + \alpha\hat{v}(t) + \delta\varepsilon(t) \tag{3}$$

$$A\hat{v}(t) = Bu(t-1) + C\varepsilon(t) \tag{4}$$

Substituting for $\hat{v}(t)$ in Eqn. (3) using Eqn. (4) gives:

$$A'\hat{y}(t+d) = B'u(t-1) + C'\varepsilon(t) \tag{5}$$

where,

$$A' = A,\quad B' = A\beta + \alpha B,\quad C' = A\delta + \alpha C$$

Equation (5) has an ARMAX type model structure and can therefore be used to design a predictive controller with inferential capabilities. The use of a single model for both the controller and inferential estimator has a number of advantages including reduced computational overheads and reduced scope for process-model mismatch [5]. It also alleviates the problems of 'interactions' between separate estimator and controller if both were to be implemented adaptively.

Inferential Predictive Control Law

Using Eqn. (5) and the cost function,

$$\min J = \sum_{j=N1}^{N2}\left[y(t+j)-r(t+j)\right]^2 + \lambda\sum_{j=1}^{Nu}\left[\Delta u(t+j-1)\right]^2 \quad (6)$$

and applying the GPC design procedure [8] leads to the following control law:

$$\mathbf{u}_t = (\mathbf{G}^T\mathbf{G}+\lambda\mathbf{I})^{-1}[\mathbf{G}^T(\mathbf{r}-\mathbf{f})+\lambda\mathbf{u}_{t-1}] \quad (7)$$

where: $\mathbf{r} = [r(t+1), r(t+2), ..., r(t+N2)]^T$

$\mathbf{f} = [f(t+1), f(t+2), ... f(t+N2)]^T$

$f(t+j) = F_j y(t) + \hat{G}_j u(t-1)$

Here $\mathbf{u}_{t-1}$ is a vector of the previously calculated controls, **f** is the free response, and **r** is a vector of the future set points. The prediction error acts to remove any offset due to quantifiable process-model mismatch. **G** is analogous to the original version of GPC and λ is a weighting factor that penalises excessive changes in the manipulated input. *N1* and *N2* are the minimum and maximum prediction horizons respectively, while *Nu* is the control horizon. As only the first calculated control move is implemented, the control law (Eqn. 7) becomes,

$$u(t) = \mathbf{h}(\mathbf{r}-\mathbf{f}) + \mathbf{g}\lambda\mathbf{u}_{t-1} \quad (8)$$

where $\mathbf{h}$ = first row of $(\mathbf{G}^T\mathbf{G}+\lambda\mathbf{I})^{-1}\mathbf{G}^T$

$\mathbf{g}$ = first row of $(\mathbf{G}^T\mathbf{G}+\lambda\mathbf{I})^{-1}$

ROBUSTNESS ISSUES AND CONTROLLER TUNING

The inferential process model, Eqn. (5), may be subject to errors and this must be taken into account when tuning the inferential predictive controller. Here, robust control philosophies are used to develop an appropriate controller parameter selection procedure. The aim is to design a controller capable of maintaining satisfactory system performance in the presence of a prespecified degree of model uncertainty.

Uncertainty Description

The uncertainty description is used to define a 'family' of plants around the nominal model. If the system is stable for all plants, *p*, then it is said to be *robustly stable*. Mathematically the family of plants, Π, can be defined as:

$$\Pi = \left\{p : \left|p(i\omega) - \tilde{p}(i\omega)\right| \le \bar{l}_a(\omega)\right\} \text{ and } \bar{l}_m(\omega) = \frac{\bar{l}_a(\omega)}{\left|\tilde{p}(i\omega)\right|} \quad (9)$$

where $p(i\omega)$ and $\tilde{p}(i\omega)$ are the actual and nominal process models respectively; $\bar{l}_a(\omega)$ is the maximum additive uncertainty for a given frequency, ω. $\bar{l}_m(\omega)$ is the corresponding maximum multiplicative uncertainty.

With digital control systems, the effect of sampling upon the system uncertainty has to be considered. Morari and Zafirou [9] have shown that the uncertainty bounds for the discrete time model are related to the continuous process according to,

$$\Pi^* = \left\{p(s) : \left|p_\gamma^*(e^{i\omega T}) - \tilde{p}_\gamma^*(e^{i\omega T})\right| \le \bar{l}_a^*(\omega)\right\} \quad (10)$$

and

$$\bar{l}_a^*(\omega) = \left|p_\gamma^*(e^{i\omega T}) - \tilde{p}_\gamma^*(e^{i\omega T})\right| \le \frac{1}{T}\left|h_o\gamma(i\omega + ik\omega_s)\right|\bar{l}_a(\omega + k\omega_s) \quad (11)$$

where *T* is the sample time, ω_s the sampling frequency, $h_o(s)$ represents a zero-order hold and $\gamma(s)$ an anti-aliasing filter. The superscript * denotes a discrete quantity. The inter-sample behaviour is modelled as a zero-order hold. This is satisfactory for chemical process systems where the disturbances are usually of low frequency, characterised by infrequently occurring random changes of at least the period of the process time constant [10,11]. As a consequence of Eqn. (10), any plant, *p(s)*, belonging to the continuous time set of plants, Π, also belongs to Π^*. The uncertainty is calculated from the nominal model by assuming an error in the phase and amplitude ratio. Calculation of the additive uncertainty is relatively straightforward once the uncertainty bound has been established.

Sensitivity and Complementary Sensitivity Functions

Consider the sampled data two-degree-of-freedom Internal Model Control (IMC) strategy shown in Fig. 1 [9]. Solid lines represent discrete data flow while the dotted lines represent continuous data flow. As will be shown later, the inferential predictive controller can also be represented in this form.

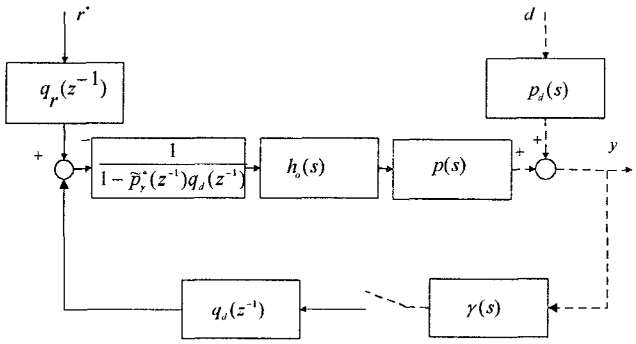

Figure 1. Two-degree-of-freedom IMC feedback structure

The closed loop system can be represented in terms of the filtered process output, y_γ^*, where $p_\gamma = p.\gamma$ and $p_{d\gamma} = p_d.\gamma$.

$$y_\gamma^* = \frac{p_\gamma^*(z^{-1})q_r(z^{-1})}{1+q_d(z^{-1})\left(p_\gamma^*(z^{-1})-\tilde{p}_\gamma^*(z^{-1})\right)}r^* + \frac{p_{d\gamma}^*(z^{-1})\left(1-p_\gamma^*(z^{-1})q_d(z^{-1})\right)}{1+q_d(z^{-1})\left(p_\gamma^*(z^{-1})-\tilde{p}_\gamma^*(z^{-1})\right)}d \quad (12)$$

Defining $e^* = r^* - y_\gamma^*$,

$$e^* = \left(1 - \frac{p_\gamma^*(z^{-1})q_r(z^{-1})}{1+q_d(z^{-1})\left(p_\gamma^*(z^{-1})-\tilde{p}_\gamma^*(z^{-1})\right)}\right)r^* - \frac{p_{d\gamma}^*(z^{-1})\left(1-p_\gamma^*(z^{-1})q_d(z^{-1})\right)}{1+q_d(z^{-1})\left(p_\gamma^*(z^{-1})-\tilde{p}_\gamma^*(z^{-1})\right)}d \quad (13)$$

Assuming that in the region of interest, the nominal plant, $\tilde{p}_\gamma^*(z^{-1})$, is an accurate representation of the real process, $p_\gamma^*(z^{-1})$, i.e. $p_\gamma^*(z^{-1}) = \tilde{p}_\gamma^*(z^{-1})$, then,

$$e^{*}=\left(1-\tilde{p}_{\gamma}^{*}(z^{-1})q_{r}(z^{-1})\right)r^{*}-p_{d\gamma}^{*}(z^{-1})\left(1-\tilde{p}_{\gamma}^{*}(z^{-1})q_{d}(z^{-1})\right)d \qquad (14)$$

For servo and regulatory control respectively, the sensitivity functions, ε_r^* and ε_d^*, and complementary sensitivity functions, η_r^* and η_d^*, are defined as,

$$\varepsilon_r^{*}=\frac{e^{*}}{r^{*}}=1-\tilde{p}_{\gamma}^{*}(z^{-1})q_{r}(z^{-1}) \qquad (15)$$

$$\eta_r^{*}=1-\varepsilon_r^{*}=\tilde{p}_{\gamma}^{*}(z^{-1})q_{r}(z^{-1}) \qquad (16)$$

$$\varepsilon_d^{*}=-\frac{e^{*}}{d}=p_{d\gamma}^{*}(z^{-1})\left(1-\tilde{p}_{\gamma}^{*}(z^{-1})q_{d}(z^{-1})\right) \qquad (17)$$

$$\eta_d^{*}=1-\varepsilon_d^{*}=1-p_{d\gamma}^{*}(z^{-1})\left(1-\tilde{p}_{\gamma}^{*}(z^{-1})q_{d}(z^{-1})\right) \qquad (18)$$

To achieve *robust performance,* the controller must be tuned appropriately. For disturbance rejection the criterion is:

$$\left|\varepsilon_d^{*}w_d\right|+\left|\eta_d^{*}\bar{l}_m^{*}\right|<1 \qquad (19)$$

Under servo control the prefilter, $q_r(z^{-1})$, does not feature in the feedback path and has no effect on stability. It can therefore be designed exclusively for robust tracking performance giving:

$$\left|1-\tilde{p}_{\gamma}^{*}(z^{-1})q_{r}(z^{-1})w_{r}\right|+\left|\tilde{p}_{\gamma}^{*}(z^{-1})q_{d}(z^{-1})\bar{l}_m^{*}\right|\left[1+\left|1-\frac{q_r(z^{-1})}{q_d(z^{-1})}\right|w_r\right]<1 \qquad (20)$$

Here w_d and w_r are frequency dependant performance weights specified by the user.

Directly Applied Inferential Predictive Control

Predictive controllers can be represented in two-degree-of freedom feedback form [12], as shown in Fig. 2.

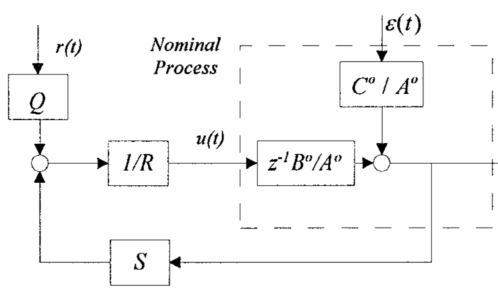

Figure 2. Two-degree-of-freedom feedback controller.

The polynomials R and S are selected for disturbance rejection while the pre-filter, Q is designed for setpoint tracking. The closed loop expression for this system is:

$$y(t)=\frac{z^{-1}B^{o}Q}{A^{o}R+z^{-1}B^{o}S}r(t)+\frac{A^{o}R}{A^{o}R+z^{-1}B^{o}S}\varepsilon(t) \qquad (21)$$

Expansion of the predictive control law (Eqn.7) and substitution into the nominal plant model,

$$A^{o}y(t)=z^{-1}B^{o}u(t)+C^{o}\varepsilon(t) \qquad (22)$$

where ^{o} denotes the true plant parameters, yields the closed-loop expression for the predictive controller,

$$y(t)=\frac{z^{-1}B^{o}C\sum h_j}{A^{o}C-z^{-1}A^{o}Cg\lambda+z^{-1}B^{o}\sum h_jF_j+z^{-1}A^{o}\sum h_j\hat{G}_j}r(t)+\frac{C^{o}\left[C-z^{-1}A^{o}Cg\lambda+z^{-1}\sum h_j\hat{G}_j\right]}{A^{o}C-z^{-1}A^{o}Cg\lambda+z^{-1}B^{o}\sum h_jF_j+z^{-1}A^{o}\sum h_j\hat{G}_j}\varepsilon(t) \qquad (23)$$

Comparison of terms between Eqns. (21) and (23) yields the polynomials for the predictive controller, with

$$Q=C\sum h_j,\ R=C-z^{-1}Cg\lambda+z^{-1}\sum h_j\hat{G}_j,\ S=\sum h_jF_j \qquad (24)$$

The sampled data IMC control terms are found by comparison of Eqns. (12) and (23):

$$q_r(z^{-1})=\frac{A^{o}Q}{A^{o}R+z^{-1}B^{o}S},\quad q_d(z^{-1})=\frac{A^{o}S}{A^{o}R+z^{-1}B^{o}S} \qquad (25)$$

Cascade Inferential Predictive Control

Industrial control configurations often adopt the cascade control structure. This is particularly common in distillation column control where typically, a product composition is measured and fed to a composition controller whose output serves as the setpoint of a tray temperature controller. The scheme is most effective where the disturbance affects both the primary and secondary variables in a similar manner. Rejecting the disturbance from the secondary loop is not always sufficient to bring the primary variable back to setpoint. Control performance suffers, due to the long cycle time of the analyser, until the secondary setpoint can be adjusted accordingly. This can be overcome if the predictive inferential controller is used in the outer loop of the cascade, because of the faster feedback of information, as illustrated in Fig. 3.

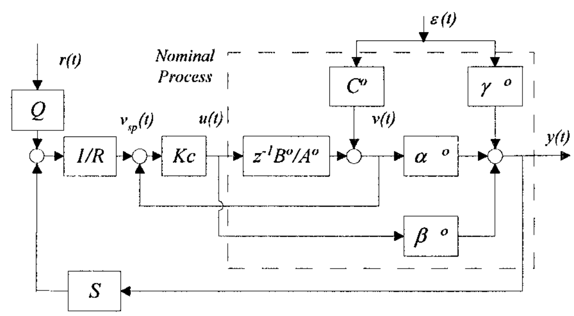

Figure 3. Inferential controller in cascade control framework.

The closed-loop expression for this control configuration is,

$$y(t)=\frac{z^{-1}Q_gB_g}{A_gA^{o}+z^{-1}S_gB_g}r(t)+\frac{\gamma^{o}A^{o}A_g}{A_gA^{o}+z^{-1}S_gB_g}\varepsilon_1(t)+\frac{\alpha^{o}C^{o}A_g-z^{-1}B_gC_g}{A_gA^{o}+z^{-1}S_gB_g}\varepsilon_2(t) \qquad (26)$$

where $A_g=A^{o}+z^{-1}K_cB^{o}$, $B_g=\alpha^{o}B^{o}+A^{o}\beta^{o}$, $C_g=K_cC^{o}$,

$$S_g=\frac{A^{o}K_cS}{R},\ Q_g=\frac{A^{o}K_cQ}{R}$$

The polynomials R, S and T are as defined previously. Similarly, the IMC filters are as defined in Eqn. (25).

Tuning of the Inferential Predictive Controllers

The flexibility of predictive controllers is due, in part, to the number of available tuning parameters. To reduce the complexity of the tuning problem, it is common practice for some default values to be attributed. For a stabilisable, detectable plant, Clarke et al [8] recommends default settings of Nu = 1, $N1$ = 1, $N2$ = 10, $\lambda = 0$. Where process time delay is significant, a prediction horizon of 10 is not always satisfactory. Lee et al [13] present a simple result, showing that a prediction horizon, $N2$ = 10 + m, where m is the process time delay, gives a closed-loop with identical poles to the delay free process with a prediction horizon, $N2$ = 10. This latter approach is used. The control weighting, λ, is used to penalise changes in the manipulated variable, de-gaining the controller and ensuring practicable application. It is common practice for the identified process noise model, $C'(z^{-1})$, to be replaced by a user defined polynomial. This design polynomial, $T(z^{-1})$, can be used to tailor the response of the system to disturbances [8]. It is especially useful in the regulatory robust design procedure as it has no effect upon the servo response of the system. The controller can therefore be designed independently for servo and regulatory control. The procedure is summarised below:

1. Identify nominal plant model
2. Design predictive controller based upon nominal plant and default tuning parameters
3. Decompose the controller into a two-degree-of-freedom IMC structure
4. Define the uncertainty bounds, $\bar{l}_a^*(\omega)$
5. Calculate $q_d(z^{-1})$ and $q_r(z^{-1})$ from Eqns. (24) and (25)
6. Calculate the sensitivity and complementary sensitivity functions for both regulatory and servo control (Eqns. 15 to 18)
7. Check if Eqn. (20) is satisfied. If not, alter control weighting, λ, and return to step 2.
8. Check if Eqn. (19) is satisfied. If not, alter design polynomial, T, and return to step 2.

APPLICATIONS TO A NON-LINEAR DISTILLATION COLUMN MODEL

The inferential predictive controllers were applied to a detailed non-linear simulation of the 8-tray pilot scale column at the University of Alberta, Canada. The column, shown schematically in Fig. 4, separates a 50/50 wt % mixture of methanol and water at a feed rate of 1.08 kg/min. In normal operation, reflux and steam flows are manipulated to control the top and bottom compositions to 95 wt % methanol and 5 wt % methanol, respectively. Bottoms composition is measured on-line via a gas chromatograph, giving a new measurement every 4 minutes. All other variables (e.g. pressures, flows, temperatures) are sampled at 30 second intervals. This model has been used for the analysis of advanced control schemes by a number of workers [5,14]. The objective here is to regulate bottoms composition when a step disturbance, lasting for approximately 3 hours, is introduced in the feed flowrate.

The feed flowrate affects column temperatures as well as product compositions. Both the temperatures and product composition also exhibit direction dependent non-linearities. The secondary variable used for inferential predictive control is the temperature on tray 3, which has significant differences in dynamics to the bottom composition [1].

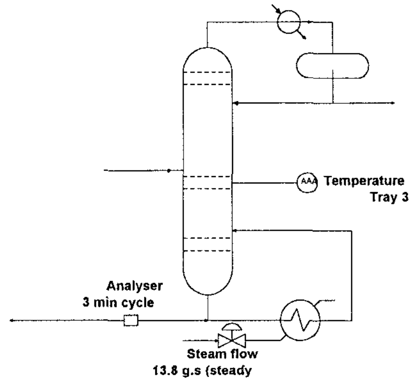

Figure 4. Schematic of pilot-scale distillation column

Employing second order estimator models, both direct and cascade inferential predictive controllers were designed for 20% error in the process parameters. For the directly applied case, the settings were $N1$ = 1, $N2$ = 10, Nu = 1, λ = 1. The settings for the cascade implementation were Kc = 2, $N1$ = 1, $N2$ = 10, Nu = 10, λ = 0.5. In both cases, offset rejection was enhanced by feeding back a filtered error as a setpoint trim.

Both inferential control strategies provided significantly better control (Figs. 5 and 6) than that achieved by a standard GPC (results not shown) as it is still limited by the large analyser delay on the primary variable. From Fig. 5, it can be seen that both control schemes offered comparable performance with both the size of the perturbations on the process output, and the settling times, being similar.

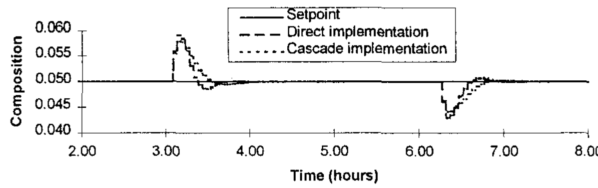

Figure 5. Comparison of direct and cascade schemes for regulatory composition control.

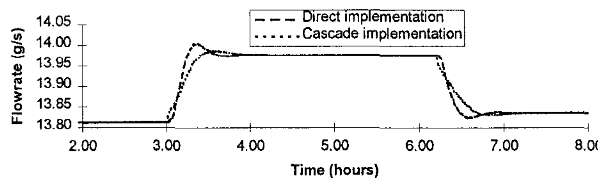

Figure 6. Manipulated variables for direct and cascade controllers.

Further simulation studies concentrated on evaluating the effectiveness of the robust tuning methodology for both implementations of the controller. In both cases, the control weighting was reduced by 20% to give control weightings of 0.8 and 0.4 respectively for the direct and cascade implementations. If the process model is accurate, control performance should be improved. This cannot be guaranteed in the presence of process-model mismatch, as in the case of this non-linear simulation. The feed disturbances were the same as that used in the previous example and the results are presented in Figs. 7 and 8.

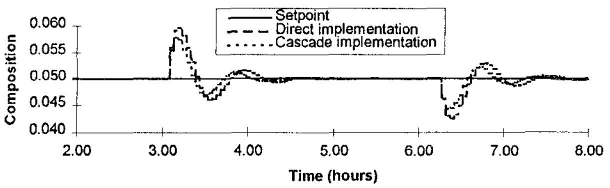

Figure 7. Comparison of non-robust inferential predictive controllers.

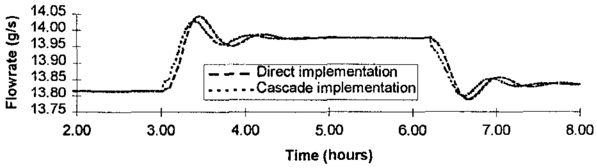

Figure 8. Manipulated variables for non-robust inferential predictive controllers.

Figure 7 clearly demonstrates degraded control performances in both the direct and cascade implementations. Larger excursions from the setpoint and more oscillatory responses with longer settling times are observed. Again, both implementations offered similar disturbance rejection properties.

The direct and cascade implementations of the inferential predictive controllers were considered for servo control and the results can be seen in Figs. 9 and 10.

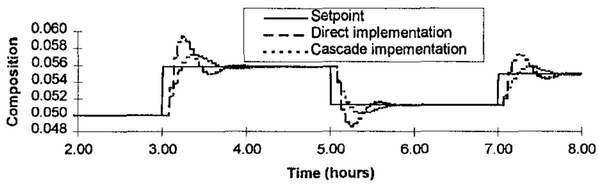

Figure 9. Comparison of direct and cascade schemes for servo control

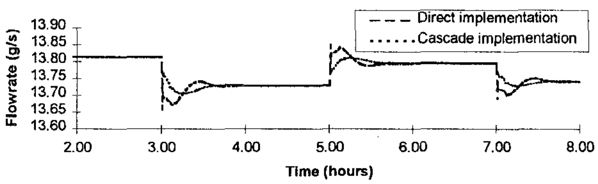

Figure 10. Manipulated variables for servo control

It is clear from Fig. 10 that the direct implementation of the controller offers superior servo control, exhibiting lower overshoot and with no offset. The manipulated variable for the cascade case is far more active than for the direct controller. The outer loop with the inferential predictive controller adjusts the temperature setpoint to a level commensurate with the desired composition. The inner loop then acts to bring the tray temperature to setpoint. This delay in reaction, between the inner loop and the inferential predictive controller, is postulated as the reason why the direct controller offers superior servo performance.

CONCLUSIONS

This paper has shown how an integrated approach to inferential predictive controller design can be developed. Two inferential control configurations were investigated; direct application of the inferential predictive control algorithm and using the predictive controller in the outer loop of a parallel cascade scheme. Both implementation offer comparative regulation of the bottoms composition of the distillation column under study, while the direct implementation offered a better servo response. The tuneable parameters of the predictive controllers were selected using a procedure based upon robust control principles. For both cases, the robust design procedure resulted in better performance compared to a more aggressively tuned regime.

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