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Inferential Feedforward Control of a Distillation Column

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Abstract

An inferential feedforward control strategy is developed and applied to a simulated distillation column. In this control strategy, the effects of disturbances on the primary process variables (top and bottom compositions) are inferred from uncontrolled secondary process variables (tray temperatures) which can be easily measured. The proposed strategies are particularly useful when disturbances cannot be measured easily or economically. Robustness of the inferential feedforward controllers and the selection of appropriate secondary measurements are discussed. Nonlinear dynamic simulation results demonstrate the superior performance of this control strategy and verify the robustness analysis.

Keywords: Feedforward control, disturbance rejection, inferential control, distillation composition control.

1. Introduction

The primary function of a process control system is to maintain the controlled process variables at their desired values in the presence of disturbances. Process plants can have large time constants and long time delays. Substantial measurement delays in some process variables such as concentration often exist. The effects of disturbances may therefore not be satisfactorily rejected through feed back control only. A strategy widely used in process control is feedforward control (Shinskey, 1979) where disturbances are measured and anticipatory control actions are taken before the controlled variables are actually affected.

In many situations, however, some disturbances cannot be easily measured. Therefore it is not possible to apply direct feedforward control in connection with these disturbances. However, in most process plants there are usually some easily measured secondary process variables, which may or may not be controlled. The correlation between disturbances and the uncontrolled secondary process variables makes it possible to infer the effects of disturbances from the measurements of these uncontrolled variables. If the secondary process variables are controlled, then the changes in their associated manipulated variables can be used to infer the effects of disturbances. Based on these inferred disturbances, feedforward control can be implemented indirectly. Yu and co-workers (Yu, 1988; Shen and Yu, 1990; 1992) proposed an indirect feedforward control strategy where the effects of disturbances are inferred from changes in the uncontrolled secondary process variables. McAvoy *et al.* (1996) propose a nonlinear inferential cascade control strategy to consider the nonlinear aspects in many industrial processes. This paper presents an inferential feedforward control strategy for a distillation column. The effects of disturbances on the top and bottom product compositions are inferred from the measurements of tray temperatures. This approach shares some common ideas with the indirect feedforward control strategy of Yu and co-workers (Yu, 1988; Shen and Yu, 1990; 1992). However, the robustness issues, which are not addressed by them, are discussed here.

The paper is organised as follows. Section 2 presents the inferential feedforward control strategy and its robustness analysis. A procedure for secondary measurement selection taking into account of the robustness issues is also presented. Section 3 describes the applications of this control strategy to a comprehensive nonlinear simulator of a methanol-water separation column. The final section contains some concluding remarks.

2. Inferential feedforward control

2.1 Feedforward control

A feedforward control system is shown in Figure 1, where disturbances are measured and compensating control actions are taken through the feedforward controller. Deviations in the controlled variables can be calculated as

$$\Delta y = GF\Delta d + G_d\Delta d \qquad (1)$$

where G is the process transfer function model, G_d is the

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disturbance model, F is the feedforward controller, u is a vector of the manipulated variables, and d is a vector of disturbances.

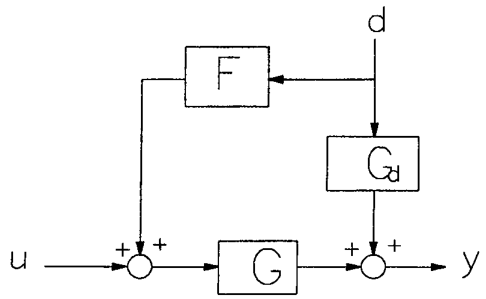

Figure 1. A feedforward control system

In order to make Δy zero, a feedforward controller of the following form is designed.

$$F = -G^{-1}G_d \tag{2}$$

In process plant applications, F is typically chosen to be of static form, and it is this approach that will be adopted in this work. To use this feedforward control strategy, the disturbances d have to be measured.

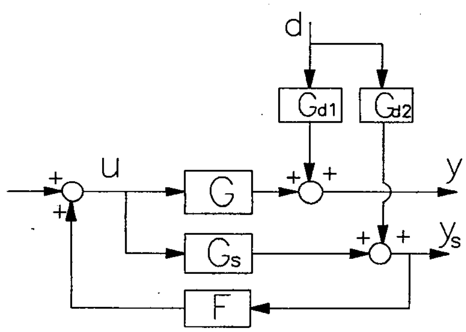

Figure 2. Inferential feedforward control

2.2 Inferential feedforward control using secondary process variable measurements

2.2.1 The structure

The framework of this control strategy is shown in Figure 2, where y represents the primary controlled variables and y_s the secondary process variables, d is a vector of unmeasured disturbances, and F is the inferential feedforward controller. The feedforward control actions can be calculated as

$$\Delta u = F\Delta y_s = F(G_{d2}\Delta d + G_s\Delta u) \tag{3}$$

Rearranging Eq(3) gives

$$\Delta u = (I - FG_s)^{-1}FG_{d2}\Delta d \tag{4}$$

The deviations in the primary process variables are given by

$$\begin{aligned}\Delta y &= G\Delta u + G_{d1}\Delta d \\ &= \{G(I - FG_s)^{-1}FG_{d2} + G_{d1}\}\Delta d\end{aligned} \tag{5}$$

To make Δy zero, it is required that

$$G(I - FG_s)^{-1}FG_{d2} + G_{d1} = 0 \tag{6}$$

Pre-multiplying both sides of Eq(6) by G^{-1} and rearranging it give the following equation.

$$(I - FG_s)^{-1}FG_{d2} = -G^{-1}G_{d1} \tag{7}$$

Denoting $(I\text{-}FG_s)^{-1}F$ by A, Eq(7) then becomes

$$AG_{d2} = -G^{-1}G_{d1} \tag{8}$$

If G_{d2} is square and non-singular, then there is only one A which solves Eq(8). If G_{d2} has more independent columns than independent rows, then there are many As which can solve Eq(8). If G_{d2} has more independent rows than independent columns, then no A will solve Eq(8). Here, we chose A to be

$$A = -G^{-1}G_{d1}G_{d2}^{+} \tag{9}$$

where $(\bullet)^+$ denotes the pseudo-inverse of $(\bullet)$ (Strang, 1980).

If G_{d2} is square and non-singular, then its pseudo-inverse is the standard inverse and the A determined by Eq(9) is the unique solution to Eq(8). If G_{d2} has more independent columns than independent rows, then the A obtained from Eq(9) solves Eq(8) in the least squares sense. If G_{d2} has more independent rows than columns, then the A calculated from Eq(9) is the solution of Eq(8) having the smallest norm (Strang, 1980). A smaller matrix A, i.e. smaller $(I\text{-}FG_s)^{-1}F$, will provide increased robustness as will be shown later.

Once A is obtained, F can be determined by solving the following equation

$$(I\text{-}FG_s)^{-1}F = A \tag{10}$$

Pre-multiplying both sides of Eq(10) by $(I\text{-}FG_s)$ and rearranging it give the following equation.

$$F(I + G_sA) = A \tag{11}$$

The inferential feedforward controller F can then be obtained as

$$\begin{aligned}F &= A(I + G_sA)^{-1} \\ &= -G^{-1}G_{d1}G_{d2}^{+}(I - G_sG^{-1}G_{d1}G_{d2}^{+})^{-1}\end{aligned} \tag{12}$$

It can be seen that the feedforward controller designed according to Eq(12) will eliminate the effects of disturbances on the primary process variables. If the models are perfect, then the effects of disturbances on the primary process variables will be either completely rejected if G_{d2}

has more independent rows than independent columns or maximally rejected if G_{d2} has more independent columns than independent rows. However, model-plant mismatches exist in most process plants and, therefore, an inferential feedforward controller alone cannot completely eliminate control offsets. It should be used in conjunction with a feedback controller.

2.2.2 Controller tuning

The addition of an inferential feedforward controller changes the overall process model and the effects of the inferential feedforward controller need to be taken into account when the feedback controller is tuned. Deviations in y_s are given by

$$\Delta y_s = G_s(\Delta u + F\Delta y_s) + G_{d2}\Delta d \quad (13)$$

Rearranging Eq(13) gives

$$\Delta y_s = (I - G_s F)^{-1} G_s \Delta u + (I - G_s F)^{-1} G_{d2}\Delta d \quad (14)$$

Deviations in the primary process variables are

$$\begin{aligned}\Delta y &= G(\Delta u + F\Delta y_s) + G_{d1}\Delta d \\ &= G[\Delta u + F(I - G_s F)^{-1}G_s \Delta u + F(I - G_s F)^{-1} G_{d2}\Delta d] + G_{d1}\Delta d \\ &= G[I + (I - FG_s)^{-1} FG_s]\Delta u + \\ &\quad [G(I - FG_s)^{-1}FG_{d2} + G_{d1}]\Delta d \end{aligned} \quad (15)$$

Due to the effects of the inferential feedforward controller, the process model is thus changed to

$$G_N = G[I + (I - FG_s)^{-1} FG_s] \quad (16)$$

Therefore, the feedback controller should be tuned for the new model, G_N, for example, using the biggest log modulus tuning (BLT) method proposed by Luyben (1986).

From Eq(12) and Eq(16), the steady state G_N is obtained as

$$G_N(0) = G(0)[I - G^{-1}(0)G_{d1}(0)\, G_{d2}^{+}(0)G_s(0)] \quad (17)$$

2.2.3 Robustness

The studies of the effects of model uncertainties in process models and disturbance models are particularly important and it is necessary to consider the impact of model uncertainties on the inferential feedforward controller. Model-plant mismatches always exist in practice, especially when a nonlinear process is approximated by a linear model around a particular operating point. Model uncertainties are assumed here to be norm bounded additive uncertainties of the following forms

$$\overline{G} = G + \Delta, \qquad \overline{\sigma}(\Delta) \le l_1 \quad (18)$$

$$\overline{G}_s = G_s + \Delta_s, \qquad \overline{\sigma}(\Delta_s) \le l_2 \quad (19)$$

$$\overline{G}_{d1} = G_{d1} + \Delta_1, \qquad \overline{\sigma}(\Delta_1) \le l_3 \quad (20)$$

$$\overline{G}_{d2} = G_{d2} + \Delta_2, \qquad \overline{\sigma}(\Delta_2) \le l_4 \quad (21)$$

where $\overline{G}$, $\overline{G}_s$, $\overline{G}_{d1}$, and $\overline{G}_{d2}$ are the true models; G, G_s, G_{d1}, and G_{d2} are the corresponding nominal models; Δ, Δ_s, Δ_1, and Δ_2 are the corresponding model uncertainties; l_1, l_2, l_3, and l_4 are the corresponding uncertainty bounds; and $\overline{\sigma}$ denotes the maximum singular value of a matrix.

Deviations in the primary process variables can be calculated from Eq(5) as

$$\begin{aligned}\Delta y &= \{(G + \Delta)(I - F(G_s + \Delta_s))^{-1}F(G_{d2} + \Delta_2) + G_{d1} + \Delta_1\}\Delta d \\ &= \{(G + \Delta)(I - FG_s - F\Delta_s)^{-1}F(G_{d2} + \Delta_2) + G_{d1} + \Delta_1\}\Delta d \\ &= \{(G + \Delta)(I - (I - FG_s)^{-1}F\Delta_s)^{-1}(I - FG_s)^{-1}F(G_{d2} + \Delta_2) \\ &\quad + G_{d1} + \Delta_1\}\Delta d \\ &= \{(I + G^{-1})G(I - (I - FG_s)^{-1}F\Delta_s)^{-1}(I - FG_s)^{-1}FG_{d2} + G_{d1}\}\Delta d \\ &\quad + \{(I + G^{-1})G(I - (I - FG_s)^{-1}F\Delta_s)^{-1}(I - FG_s)^{-1}F\Delta_2 + \Delta_1\}\Delta d \\ &= \{(I + G^{-1})G(I - (I - FG_s)^{-1}F\Delta_s)^{-1}(I - FG_s)^{-1}FG_{d2} + G_{d1}\}\Delta d \\ &\quad + \{(I + G^{-1})G(I - FG_s)^{-1}F(I - \Delta_s(I - FG_s)^{-1}F)^{-1}\Delta_2 \\ &\quad + \Delta_1\}\Delta d \end{aligned} \quad (22)$$

In order to make the effects of model uncertainties small, both $(I - FG_s)^{-1}F$ and $G(I - FG_s)^{-1}F$ should be small. Eq(9) and Eq(10) indicate that

$$(I - FG_s)^{-1}F = -G^{-1}G_{d1}G_{d2}^{+} \quad (23)$$

Thus, the effects of model uncertainties will be small if both $\overline{\sigma}(G^{-1}G_{d1}G_{d2}^{+})$ and $\overline{\sigma}(G_{d1}G_{d2}^{+})$ are small.

2.2.4 Secondary measurement selection

The inferential feedforward controller is usually designed for specific disturbances. Suppose that d' is a different disturbance which was not considered in the designing of the inferential controller. The disturbance gains from d' to the primary process variables are G_{d1}', whilst those to the secondary process variables are G_{d2}'. The deviations in the primary process variables caused by d' can be calculated from Eq(5) as

$$\begin{aligned}\Delta y &= \{G(I - FG_s)^{-1}FG_{d2}' + G_{d1}'\}\Delta d' \\ &= \{G_{d1}' - G_{d1}\, G_{d2}^{+}\, G_{d2}'\}\Delta d' \end{aligned} \quad (24)$$

It is then desirable that $G_{dN} = \{G_{d1}' - G_{d1}\, G_{d2}^{+}\, G_{d2}'\}$ be small.

There may be a number of secondary process variables that can be measured. Secondary measurements can be selected so as to minimise the effects of model uncertainties and unconsidered disturbances. Shen and Yu (1990) propose a procedure where secondary measurements are selected so as to reduce the effects of unconsidered disturbances. Here we argue that robustness is also an important factor to be considered in selecting secondary measurements.

Eq(16) indicates that the selection of secondary measurements will also affect the overall process model. If the primary controllers used are diagonal controllers, then it would be desirable that the off diagonal elements in the process model are relatively small so that control loop interactions are small. Since the objective of employing a feedforward controller is to counteract disturbances as soon as possible, the secondary measurements should have fast dynamic response to the considered disturbances so that the presence of a disturbance can be sensed quickly. The followings are therefore some factors that ought to be considered when selecting secondary measurements.

1). The selected secondary measurements have fast responses to the considered disturbances;

2). $\bar{\sigma}(G^{-1}G_{d1}G_{d2}^{+})$ is small;

3). $\bar{\sigma}(G_{d1}G_{d2}^{+})$ is small;

4). Elements of $\{G_{d1}' - G_{d1}G_{d2}^{+}G_{d2}'\}$ are small;

5). Off diagonal elements of $G[I - G^{-1}G_{d1}G_{d2}^{+}]$ are relatively small if diagonal controllers are used.

3. Application to a distillation column

3.1 The distillation process

The distillation column studied in this paper is a comprehensive nonlinear simulation of a methanol-water separation column. A nonlinear tray by tray dynamic model has been developed using mass and energy balances. This simulation has been validated against pilot plant tests and is well known for its use in control system performance studies (Tham *et al*, 1991a; 1991b). The following assumptions are imposed: negligible vapour holdup, perfect mixing in each stage and constant liquid holdup. The nominal operation data for this column are listed in Table 1.

Table 1. Nominal distillation column operation data

No. of theoretical stages	10
Feed tray	5
Feed composition (z)	50% methanol
Feed flow rate (F)	18.23 g/s
Top composition (y)	95% methanol
Bottom composition (x)	5% methanol
Top product rate (D)	9.13 g/s
Bottom product rate (B)	9.1 g/s
Reflux rate (L)	10.0 g/s
Steam rate (V)	13.8 g/s

The process model and the disturbance model for the *LV* configuration are obtained through the application of a series of step response tests. In the *LV* configuration, the top composition y is controlled by the reflux flow rate L and the bottom composition x is controlled by the steam flow rate to the reboiler V. The condenser level is controlled by the top product flow rate and the reboiler level is controlled by the bottom product flow rate. Product compositions are measured and it is assumed that there is a five minute time delay in the composition analysers. Disturbances considered here are feed rate disturbances and feed composition disturbances. The sampling interval is one minute. For the purpose of control system synthesis, the process is approximated by the following linear model:

$$\begin{pmatrix} \Delta y \\ \Delta x \end{pmatrix} = G(s)\begin{pmatrix} \Delta L \\ \Delta V \end{pmatrix} + G_{d1}(s)\begin{pmatrix} \Delta F \\ \Delta z \end{pmatrix} \qquad (25)$$

To identify the transfer function models $G(s)$ and $G_{d1}(s)$, step changes in reflux flow, steam flow to the reboiler, feed flow rate, and feed composition were imposed on the column and the resulting process data recorded. The process and disturbance models are approximated by first order lag plus delay models, in line with a number of previous studies using this column. Since the distillation column exhibits some degrees of nonlinearity, a series of positive and negative step changes are applied. From these plant test data, linear discrete time models were identified using least squares regression and converted into continues time transfer function models.

The identified process model is

$$G(s) = \begin{bmatrix} \frac{1.09e^{-5s}}{5.51s+1} & \frac{-1.30e^{-5s}}{13.72s+1} \\ \frac{2.27e^{-5s}}{17.15s+1} & \frac{-7.18e^{-5s}}{29.50s+1} \end{bmatrix} \qquad (26)$$

and the identified disturbance model is

$$G_{d1}(s) = \begin{bmatrix} \frac{0.34e^{-5s}}{89.29s+1} & \frac{10.85e^{-5s}}{15.43s+1} \\ \frac{2.64e^{-5s}}{16.67s+1} & \frac{70.26e^{-5s}}{26.25s+1} \end{bmatrix} \qquad (27)$$

Dynamic models for tray temperatures were also identified. Here only the gains are given in Table 2.

3.2 Inferential feedforward control using tray temperatures

In this study, the disturbances considered are feed rate and feed composition disturbances. In this case, two tray temperature measurements are necessary and sufficient to make Eq(8) having a unique solution. Here we use two tray temperatures to design the inferential feedforward controller. Computations of $\bar{\sigma}(G^{-1}G_{d1}G_{d2}^{+})$, $\bar{\sigma}(G_{d1}G_{d2}^{+})$, G_N, and the relative gain (λ_{11}) were carried out for all the possible selections and some typical results are given in Table 3.

Table 2. Tray temperature gains

Tray No.	L	V	F	z
1	-2.8915	6.4034	-4.4611	-102.3443
2	-1.8574	5.9962	-3.9206	-75.4604
3	-0.8662	3.2257	-1.9184	-41.8082
4	-0.4552	1.2205	-0.7124	-28.0815
5	-0.8146	1.5265	-0.7478	-23.6853
6	-1.0418	1.3930	-0.5702	-18.2355
7	-0.9795	1.2064	-0.3962	-12.2655
8	-0.6352	0.7333	-0.2021	-6.3523

Table 3. Comparison of different tray temperature selections

Tray No.	$\bar{\sigma}(G^{-1}G_{d1}G_{d2}^{+})$	$\bar{\sigma}(G_{d1}G_{d2}^{+})$	G_N	λ_{11}
1, 4	0.1889	0.9807	0.88 -0.77 0.56 -3.23	1.18
1, 8	1.2493	5.9703	-0.05 -0.01 -2.30 -0.88	2.15
2, 7	0.5675	4.3106	0.19 -0.22 -2.31 -0.63	0.19
3, 6	0.3826	2.3840	0.46 -0.47 -0.65 -1.79	0.73
5, 6	35.6054	97.5617	0.24 -0.35 31.30 -19.2	-0.70

From the results shown in Table 3, selecting tray No. 1 and 4 is the most robust selection. Table 3 indicates that it is not advisable to select tray No. 5 and 6 as this is a very non-robust selection. The relative gains shown in Table 3 also indicate that tray No. 1 and 4 should be selected while tray No. 5 and 6 should be avoided. To verify this analysis, two inferential feedforward control schemes were developed. One uses tray No. 1 and 4 while the other uses tray No. 5 and 6.

The inferential feedforward controller based on tray No. 1 and 4 is calculated using Eq(12) as

$$F_1 = \begin{bmatrix} -0.1029 & 0.0948 \\ -0.1599 & -0.2401 \end{bmatrix} \tag{28}$$

while that based on tray No. 5 and 6 is

$$F_2 = \begin{bmatrix} 4.6478 & -7.5999 \\ 4.3878 & -8.4525 \end{bmatrix} \tag{29}$$

The two inferential feedforward controllers were implemented in conjunction with diagonal PI controllers (feedback controllers). For the purpose of comparison, a feedback controller (multi-loop PI controller) without inferential feedforward control was also developed. All the controllers were tuned using the BLT tuning method (Luyben, 1986) and the controller parameters are given in Table 4.

Table 4. Controller parameters

	Top comp.		Bottom Comp.	
	K_{c1}	T_{i1}	K_{c2}	T_{i2}
Feedback control only	0.6477	19.3399	-0.4058	23.1993
Feedback control with F_1	0.6543	18.9721	-0.3807	21.005
Feedback control with F_2	0.9747	11.1862	-0.1086	44.0574

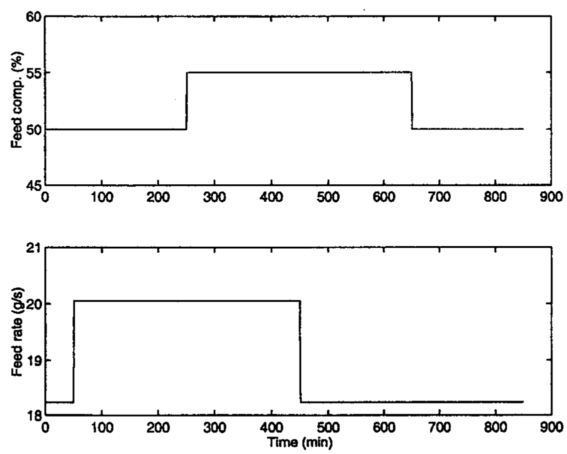

Figure 3. Disturbance sequence

To test the performance of the inferential feedforward controllers, disturbances shown in Figure 3 were applied to simulation. These disturbances represent a 10% increase in the feed rate at the 51st minutes, a 10% increase in the feed composition at the 251st minutes, a 10% decrease in the feed rate at the 451st minutes, and finally a 10% decrease in the feed composition at the 651st minutes. Figure 4 shows the performance of the three control systems. The sepoint for the top composition is 95% methanol while that for the bottom composition is 5% methanol. The solid, dashed, and dotted lines in Figure 4 represent, respectively, the responses from feedback control only, feedback control with F_1, and feedback control with F_2. The sums of squared control errors of the three control schemes are given in Table 5. It can be seen from Figure 4 and Table 5 that improved control performance has been obtained by using F_1. Using F_2 improves the top composition control performance. However, it significantly deteriorates the bottom composition control performance. This is most likely due to the fact that the control system with F_2 is not robust to model uncertainties. The simulation results verify the robustness analysis presented in this paper.

Table 5. Sum of squared control errors

Control schemes	Top comp.	Bottom Comp.
Feedback control only	23.19	195.69
Feedback control with F_1	17.47	117.85
Feedback control with F_2	2.71	3097.8

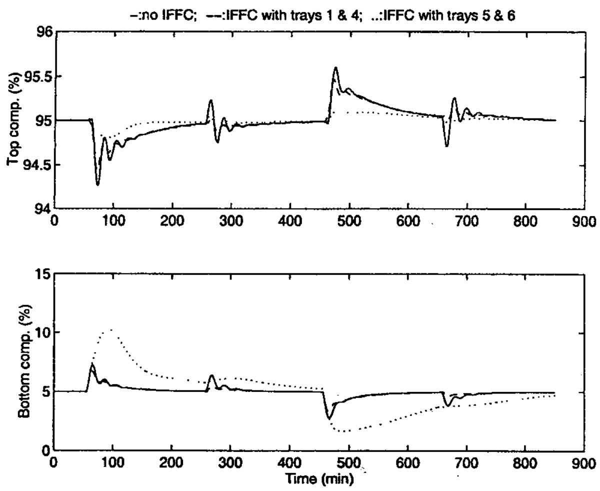

Figure 4. Control performance

5. Conclusions

An inferential feedforward control strategy is proposed and applied to distillation composition control. In this control strategy, the effects of disturbances on the primary process variables are inferred from certain easily available measurements of uncontrolled secondary process variables. This strategy is particularly useful when disturbances cannot easily be measured and, hence, direct feedforward control cannot be applied. The main advantage of such an inferential feedforward control strategy is that measurements of disturbances are not needed. Robustness analysis of the inferential feedforward control strategy is carried out and it is shown that robustness is an important factor in the selection of secondary measurements. Nonlinear dynamic simulation results show that the proposed strategies can greatly improve disturbance rejection ability of the distillation composition control system. Robustness analysis presented in this paper is also verified by the simulation results. Inferential feedforward control with multiple tray temperatures (more than two) will be studied and reported in the future.

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