

# Coupled Water Tank Experiments



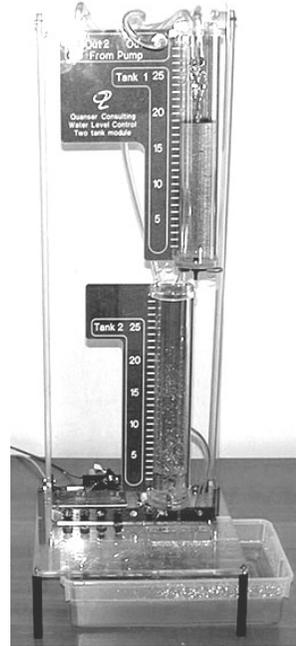
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# Coupled Water Tank Experiments

## Quanser Consulting Inc.

### 1 Description

The “Two Tank Module” consists of a pump with a water basin . The pump pumps water vertically to two **quick connect, normally closed** orifices “Out1” and “Out2”. These two orifices have different diameters. Two tanks mounted on the front plate are configured such that flow from the first tank flows into the second tank and outflow from the second tank flows into the main water basin. The outflows from the two tanks are variable by changing inserts that screw into the tapped holes. Rubber tubing with appropriate couplings are supplied such that the pump can pump into one or both tanks. The selection of outputs from the pump controls the flow ratio between the two outputs.



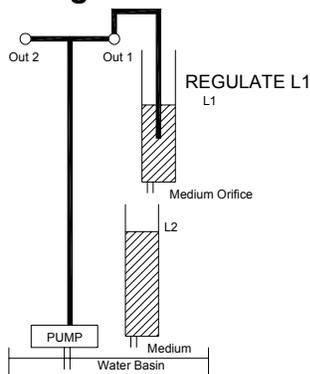
This single system can be configured into 3 types of experiments each with various parameters.

**Configuration #1:** SISO system. The pump feeds into tank1 and you design a controller that regulates the level in Tank 1. Tank2 is not used at all.

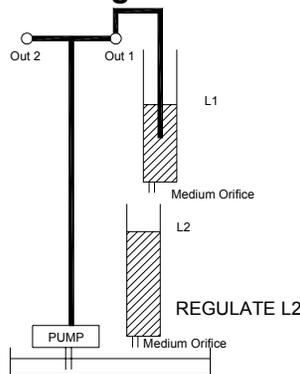
**Configuration #2:** State coupled SISO system. The pump feeds into tank1 which in turn feeds into tank 2. Design a controller that regulates the levels in tank #2. Try different orifices in tank #1 and tank #2

**Configuration #3:** State coupled and input coupled SISO system. The pump feeds into and into tank 2 using a split flow. Tank1 also feeds into tank 2. Design a controller that regulates the levels in tank #2.

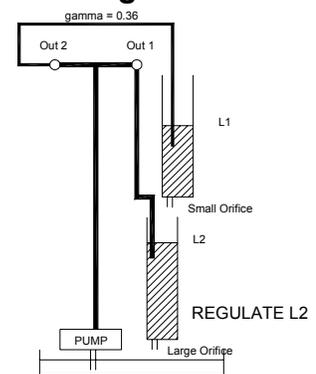
#### Configuration #1



#### Configuration #2

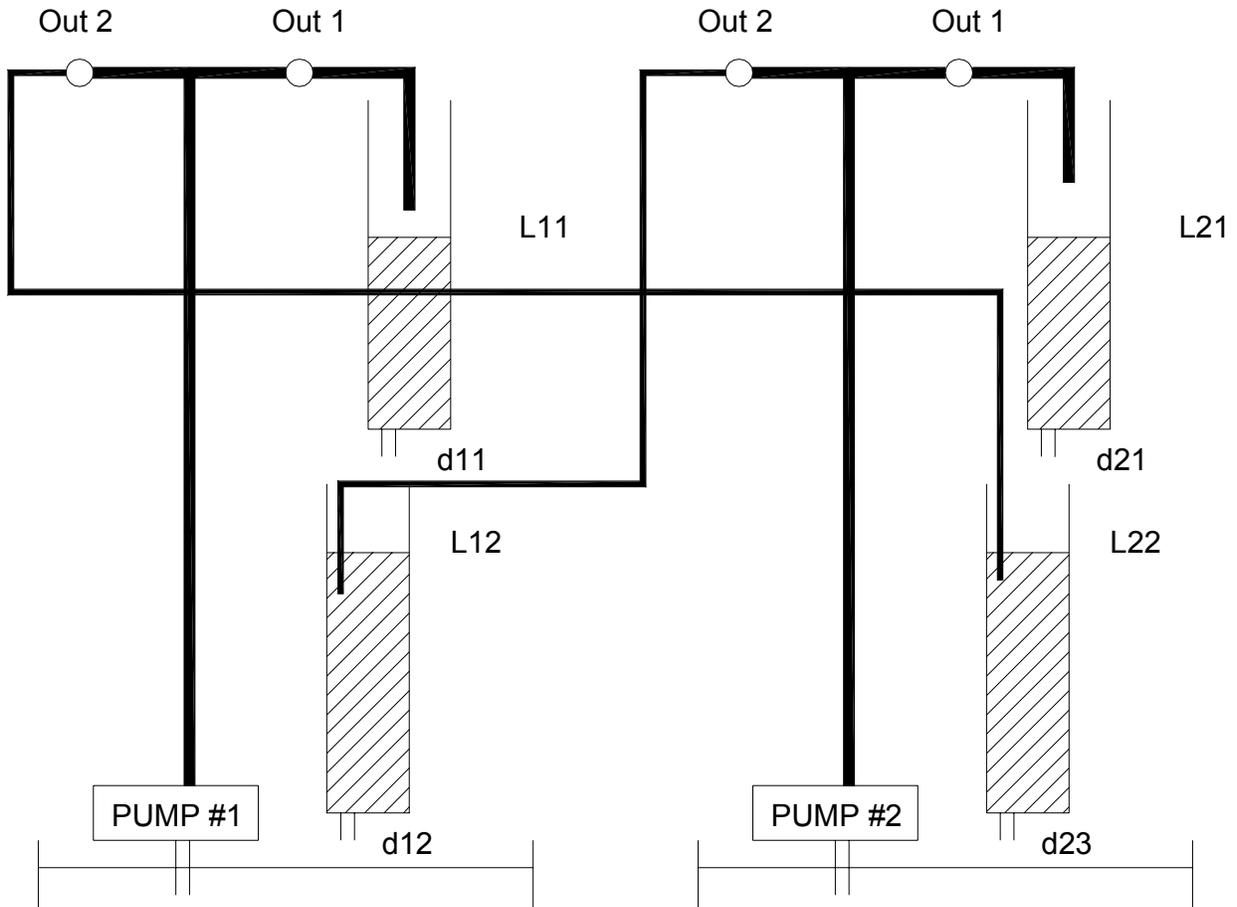


#### Configuration #3



Coupled Tanks 1

If you would like to perform more complex experiments, you may couple several Two Tank Modules as shown in Figure 5.



Coupled Tanks 2

## 2 Mathematical Models

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### 2.1 Single tank

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Consider the single tank system shown in Configuration 1.

The inflow to the tank is:

$$F_{in} = K_m V_p \text{ cm}^3/\text{sec}$$

where  $K_m$  is the pump constant and  $V_p$  is the voltage applied to the pump.

The outflow velocity is given by the Bernoulli equation for small orifices:

$$V_o = \sqrt{2 g L_1} \text{ cm/sec}$$

where  $g$  is the gravitational acceleration in  $\text{cm} / \text{sec}^2$  and  $L_1$  is the height of the water level in the tank in  $\text{cm}$ .

The outflow rate is then :

$$F_{out} = a_1 \sqrt{2 g L_1} \text{ cm}^3/\text{sec} \text{ where } a_1 \text{ is the outflow orifice diameter}$$

The flow rate through the tank is then given by

$$F_{in} - F_{out} = K_m V_p - a_1 \sqrt{2 g L_1} \text{ cm}^3/\text{sec}$$

The change in level is then given by

$$\dot{L}_1 = - \frac{a_1}{A_1} \sqrt{2 g L_1} + \frac{K_m}{A_1} V_p \text{ cm/sec}$$

where  $A_1$  is the diameter of the tank

Linearizing about a quiescent point  $L_o$  we obtain:

$$\dot{L}_1 = - \frac{a_1}{A_1} \sqrt{\frac{g}{2 L_o}} L + \frac{K_m}{A_1} V_p$$

For a given level  $L_0$ , we need a constant voltage applied to the pump in order to maintain the level. That means an integrator is required in the control loop to compensate for the constant (relatively) disturbance due to the outflow orifice. We write the final state space equations for the single tank control problem as:

$$\begin{bmatrix} \dot{L}_1 \\ \dot{\zeta}_1 \end{bmatrix} = \begin{bmatrix} -\frac{a_1}{A_1} \sqrt{\frac{g}{2L_0}} & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} L \\ \zeta_1 \end{bmatrix} + \begin{bmatrix} \frac{K_m}{A_1} \\ 0 \end{bmatrix} V_p$$

we have introduced an integrator for the the level  $\dot{\zeta}_1 = L_1$ .

The quiescent voltage required to maintain the level at  $L_0$  is obtained using:

$$0 = K_m V_{p0} - a_1 \sqrt{2 g L_0}$$

$$V_{p0} = a_1 \frac{\sqrt{2 g L_0}}{K_m}$$

## 2.2 Two tank system : State coupling only

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Consider Configuration #2 where the pump feeds Tank 1 and Tank 1 feeds Tank 2. For Tank 1 #1 the same equations developed above apply.

As for tank #2, the inflow is equal to the outflow from Tank 1:

$$F_{2_{in}} = a_1 \sqrt{2 g L_1} \text{ cm}^3/\text{sec}$$

and the outflow is:

$$F_{2_{out}} = a_2 \sqrt{2 g L_2} \text{ cm}^3/\text{sec}$$

So the level in Tank 2 is written as:

$$\dot{L}_2 = \frac{a_1}{A_2} \sqrt{2 g L_1} - \frac{a_2}{A_2} \sqrt{2 g L_2}$$

which can be linearized about  $L_{10}$  &  $L_{20}$

to obtain:

$$\dot{L}_2 = -\frac{a_2}{A_2} \sqrt{\frac{g}{2L_{20}}} L_2 + \frac{a_1}{A_2} \sqrt{\frac{g}{2L_{10}}} L_1$$

The complete system in can be written as 2 2<sup>nd</sup> order systems:

Tank 1 uses the pump voltage as input:

$$\begin{bmatrix} \dot{L}_1 \\ \dot{\zeta}_1 \end{bmatrix} = \begin{bmatrix} -\frac{a_1}{A_1} \sqrt{\frac{g}{2L_{10}}} & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} L_1 \\ \zeta_1 \end{bmatrix} + \begin{bmatrix} \frac{K_m}{A_1} \\ 0 \end{bmatrix} V_p$$

and Tank2 uses the height in L1 as an input:

$$\begin{bmatrix} \dot{L}_2 \\ \dot{\zeta}_2 \end{bmatrix} = \begin{bmatrix} -\frac{a_2}{A_2} \sqrt{\frac{g}{2L_{20}}} & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} L_2 \\ \zeta_2 \end{bmatrix} + \begin{bmatrix} \frac{a_1}{A_2} \sqrt{\frac{g}{2L_{10}}} \\ 0 \end{bmatrix} L_1$$

We can then design a two loop system to regulate the level in L2.

Note that for quiescent conditions the heights remain constant:

$$0 = \frac{a_1}{A_2} \sqrt{2g} L_1 - \frac{a_2}{A_2} \sqrt{2g} L_2$$

which when solved for  $L_{20}$  results in:

$$L_{10} = \frac{a_2^2}{a_1^2} L_{20}$$

That means could technically regulate L2 using L1 alone BUT since the ratio is not precisely known a closed loop system that feeds back L2 would improve performance.

the quiescent voltage required to maintain the level at L1 is obtained using:

$$0 = K_m V_p - a_1 \sqrt{2 g L_{10}}$$

$$V_p = a_1 \frac{\sqrt{2 g L_{10}}}{K_m}$$

### 2.3 Two tank system with state and input coupling

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Configuration #3 accommodates for the pump flow to split to tank #1 and tank #2.

Let  $o_1$  be the diameter of outflow #1 (Out1) and  $o_2$  be the diameter of outflow #2 (Out2) which are coming from the pump. Let

If you attach only one connector, the flow is controlled directly by the voltage applied to the pump. When you attach two connectors as in configuration 3 you then have two flows coming out. The total flow is still controlled by the pump and the individual flows are split up in proportion to the areas  $o_1$  and  $o_2$  as given below:

$$F_{p1} = \frac{a o_1}{a o_1 + a o_2} K_m V_p = \gamma K_m V_p \text{ cm}^3/\text{sec}$$

$$F_{p2} = \frac{a o_2}{a o_1 + a o_2} K_m V_p = (1-\gamma) K_m V_p \text{ cm}^3/\text{sec}$$

For configuration #3, we feed ***Fp1 (Out1) Directly to Tank #2*** and ***Out #2 Directly to Tank #1***.

By combining the state space equations obtained in section 2 with the above two equations we have:

$$\begin{bmatrix} \dot{L}_1 \\ \dot{L}_2 \\ \dot{\zeta}_2 \end{bmatrix} = \begin{bmatrix} -\frac{a1}{A1} \sqrt{\frac{g}{2L_{10}}} & 0 & 0 \\ \frac{a1}{A2} \sqrt{\frac{g}{2L_{10}}} & -\frac{a2}{A2} \sqrt{\frac{g}{2L_{20}}} & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \\ \zeta_2 \end{bmatrix} + \begin{bmatrix} (1-\gamma) \frac{K_m}{A_1} \\ \gamma \frac{K_m}{A_2} \\ 0 \end{bmatrix} V_p$$

For quiescence we have:

$$0 = - a_1 \sqrt{2 g L_{10}} + (1-\gamma) K_m V_p$$

and

$$0 = a_1 \sqrt{2 g L_{10}} - a_2 \sqrt{2 g L_{20}} + \gamma K_m V_p$$

which result in:

$$L_{10} = \frac{a_2^2}{a_1^2} (\gamma - 1)^2 L_{20}$$

and

$$V_{p0} = \frac{a_2}{K_m} \sqrt{2 g L_{20}}$$

### 3 Control Systems Design

Note the configurations for these experiments. You must use the appropriate exit orifice for each tank and the appropriate feed from the pump. Note that for configuration #3, Out #1 feeds to Tank #2 and Out #2 feeds to Tank #1

Configuration	$a_1$	$L_{10}$	$a_2$	$L_{20}$	Out <sub>1</sub>	Out <sub>2</sub>	Control Tank Level
1	Medium	15 cm	Medium	N/A	Tank1	N/A	$L_1$
2	Medium	Compute	Medium	15 cm	Tank1	N/A	$L_2$
<b>NOTE REVERSED FEED AND ORIFICE CHANGE IN CONFIGURATION #3</b>							
3	Small	Compute	Large	10 cm	Tank2	Tank1	$L_2$

#### 3.1 Configuration #1

The open loop system is:

$$\begin{bmatrix} \dot{L}_1 \\ \dot{\zeta}_1 \end{bmatrix} = \begin{bmatrix} -\frac{a_1}{A_1} \sqrt{\frac{g}{2L_o}} & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} L \\ \zeta_1 \end{bmatrix} + \begin{bmatrix} \frac{K_m}{A_1} \\ 0 \end{bmatrix} V_p$$

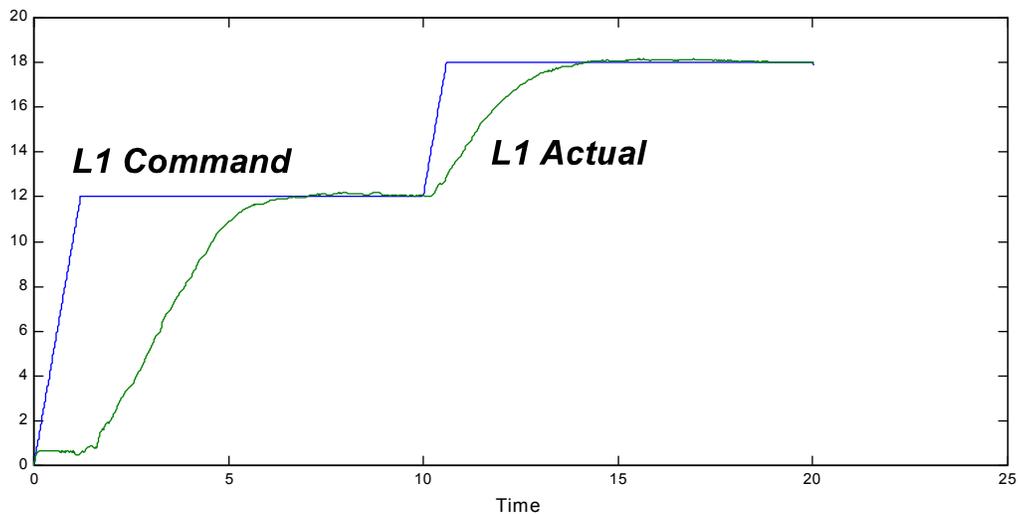
The controller is obtained by running the m file **d\_level1.m** from MATLAB which performs LQR design to obtain the feedback gains:

$$V_p = -k_{p1} (L_1 - L_{1c}) - k_{i1} \int (L_1 - L_{1c}) + \left[ a_1 \frac{\sqrt{2 g L_{1c}}}{K_m} \right]$$

where  $L_{1c}$  is the commanded level to the tank in cm. This is a PI controller that commands the voltage to the motor based on the error signal and a feedforward term (in brackets =  $V_q$  as calculated in section 2.1) that pre-computes the desired voltage for the desired level. The feedforward term ensures reduces the windup in the integrator.

The controller is implemented using SIMULINK model **q\_level1.mdl** and run in realtime using WinCon.

The response is shown in Figure below.



### 3.1 Configuration #2

The open loop system consists of two systems:

$$\begin{bmatrix} \dot{L}_1 \\ \dot{\zeta}_1 \end{bmatrix} = \begin{bmatrix} -\frac{a1}{A1} \sqrt{\frac{g}{2L_o}} & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} L \\ \zeta_1 \end{bmatrix} + \begin{bmatrix} \frac{K_m}{A_1} \\ 0 \end{bmatrix} V_p \quad \text{which is the first tank}$$

and second system which is the second tank and whose input is the level of Tank #1:

$$\begin{bmatrix} \dot{L}_2 \\ \dot{\zeta}_2 \end{bmatrix} = \begin{bmatrix} -\frac{a2}{A2} \sqrt{\frac{g}{2L_{20}}} & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} L_2 \\ \zeta_2 \end{bmatrix} + \begin{bmatrix} \frac{a1}{A2} \sqrt{\frac{g}{2L_{10}}} \\ 0 \end{bmatrix} L_1$$

The first system is controlled using the controller designed in section 3.1.

$$V_p = -k_{p1} (L_1 - L_{1c}) - k_{i1} \int (L_1 - L_{1c}) + \left[ a_1 \frac{\sqrt{2 g L_{1c}}}{K_m} \right] \text{(inner Loop)}$$

The command to that controller is derived from the controller for the second tank. The second tank controller is also designed using lqr to obtain the feedback gains for the feedback equation:

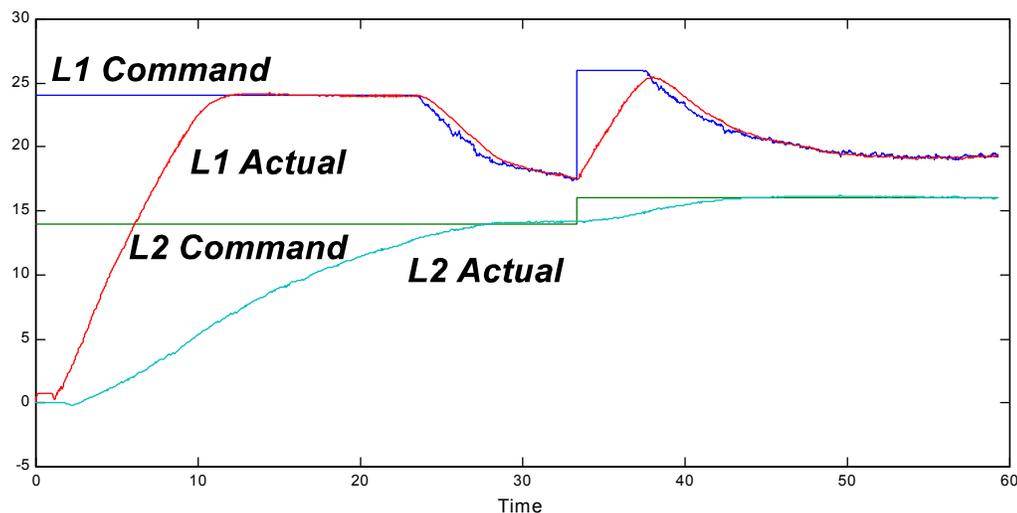
$$L_{1c} = -k_{p2} (L_2 - L_{2c}) - k_{i2} \int (L_2 - L_{2c}) + \left[ \frac{a_2^2}{a_1^2} L_{2c} \right] \text{(outer loop)}$$

The controller is obtained by running the m file **d\_level2.m** from MATLAB which performs LQR design to obtain the feedback gains.

The idea is then to control the level of Tank 1 such that tank 2 tracks the desired command. From section 2.2 we know that if the outflow orifices are the same, the steady state condition is that the two levels should be the same. It is good practice however to limit the command  $L_{1c}$  to within the bounds of the tanks so that the water does not flow out in case you have some kind of instability in the outer loop.

The controller is implemented using SIMULINK model **q\_level2.mdl** and run in realtime using WinCon.

The response is shown in Figure below.



### 3.1 Configuration #3

In this case, the two tank equations cannot be written independently since the input  $V_p$  affects both tanks directly and it is the only control input. The state space equation is:

$$\begin{bmatrix} \dot{L}_1 \\ \dot{L}_2 \\ \dot{\zeta}_2 \end{bmatrix} = \begin{bmatrix} -\frac{a_1}{A_1} \sqrt{\frac{g}{2L_{10}}} & 0 & 0 \\ \frac{a_1}{A_2} \sqrt{\frac{g}{2L_{10}}} & -\frac{a_2}{A_2} \sqrt{\frac{g}{2L_{20}}} & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \\ \zeta_2 \end{bmatrix} + \begin{bmatrix} (1-\gamma) \frac{K_m}{A_1} \\ \gamma \frac{K_m}{A_2} \\ 0 \end{bmatrix} V_p$$

Note that we introduced an integrator for  $L_2$  since we want to regulate  $L_2$ . We then design a controller of the form:

$$V_p = - \left[ k_1 (L_1) + k_2 (L_2 - L_{2c}) + k_3 \int (L_2 - L_{2c}) \right] + \left[ \frac{a_2}{K_m} \sqrt{2 g L_{2c}} \right]$$

This controller may be implemented as is but note that  $L_1$  is free and uncontrolled.

We can rewrite the feedback component as:

$$V_p = - k_1 \left[ L_1 - \left[ -\frac{k_2}{k_1} (L_2 - L_{2c}) - \frac{k_3}{k_2} \int (L_2 - L_{2c}) \right] \right]$$

$$V_p = - k_{p1} [L_1 - L_{1c}]$$

$$L_{1c} = \left[ -k_{p2} (L_2 - L_{2c}) - k_{i2} \int (L_2 - L_{2c}) \right]$$

which is a proportional + integral outer loop to control  $L_2$  by commanding  $L_{1c}$  and a proportional inner loop to control  $L_1$  to track  $L_{1c}$ . This way we can limit  $L_{1c}$  not to exceed the tank dimensions. We can also add on a quiescent feedforward term for  $L_{1c}$  derived from (see section 2.3)

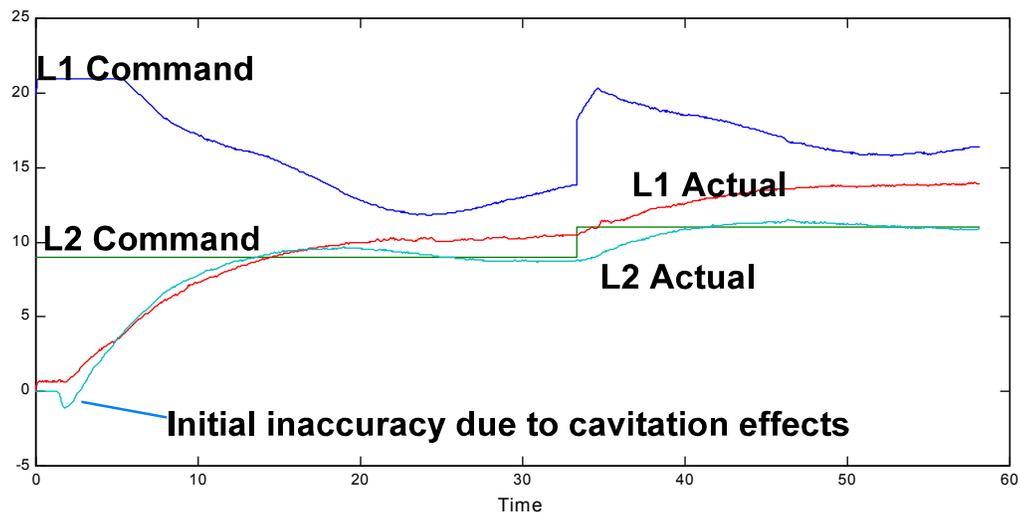
$$L_{10} = \frac{a_2^2}{a_1^2} (\gamma - 1)^2 L_{20}$$

resulting in :

$$L_{1c} = \left[ -k_{p2} (L_2 - L_{2c}) - k_{i2} \int (L_2 - L_{2c}) \right] + \frac{a_2^2}{a_1^2} (\gamma - 1)^2 L_{2c}$$

and

$$V_p = -k_{p1} [L_1 - L_{1c}] + \left[ \frac{a_2}{K_m} \sqrt{2 g L_{2c}} \right]$$



## 5 Wiring and Calibration

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**Use UPM2405 amplifier for this experiment**

From	To	Cable	Function
Level sensors	S1 & S2 on UPM2405	6 pin mini Din/ 6 pin mini Din	Measure V1 &V2 via S1 &S2
"To/ A/D" on UPM	MultiQ Analog inputs 0,1,2,3	5 pin Din 4 x RCA	V1 to A/D #0 V2 to A/D #1
MultiQ D/A # 0	UPM "From D/A"	RCA 5 Pin Din Mono	Signal to amplifier
To Load	Motor Connector	6 pin Din / 4 pin Din <b>Gain = 5 ( green marker)</b>	Power to pump

Using this wiring S1 senses V1 from Tank 1 and S2 senses V2 from Tank 2

### 5.1 Calibration

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To calibrate:

Wire the system as described and power up the amplifier.

#### **a) calibrate the zero**

Run WinCon - load project **cal\_tank.wcp** -this displays the voltages from the two sensors. With no water in the tanks adjust the **offset potentiometers** obtain 0.0 volts. Click stop when you are done and close WinCon.

#### **b) calibrate the maximum voltage**

Run WinCon - load project **tank\_cal.wcp** - this displays the voltages from the two sensors. Plug the outflow of tank 1 with your finger or use the supplied screw . Fill Tank1 up to 25 cm. Adjust the **gain potentiometer** for tank1 to obtain anywhere between **4.0 to 4.2** volts on the position sensor. Click stop when you are done and close WinCon. Release the water out of the tank. The measurement should be zero.

Repeat for Tank 2.

Click stop when you are done and close WinCon.

#### **c) Run the test controller for configuration 1**

**MAKE SURE THE TUBE FROM OUT 1 IS INSERTED INTO THE TANK1. Do not use**

**a tube to OUT 2.**

Start WinCon, load q\_level1.wcl and run it . The tank should fill up to 15 cm and maintain its level. **Be ready to stop the controller if the water level rises above 25 cm!** Check wiring and calibration if the system does not perform as expected.

**6 Software**

<b>File</b>	<b>Software needed</b>	<b>Function</b>	<b>Output /Input filenames ( I = input file, O = Output file)</b>	<b>I/O</b>
<b>d_level1.m</b>	MATLAB	Derive gains for conf 1	Workspace	I O
<b>q_level1.mdl</b>	SIMULINK WinCon	Controller diagram Implement controller	q_level1.wcl	O
<b>q_level1.wcp</b>	WinCon	Run controller	q_level1.wcl	O
<b>d_level1.m</b>	MATLAB	Derive gains for conf 1	Workspace	I O
<b>q_level1.mdl</b>	SIMULINK WinCon	Controller diagram Implement controller	q_level1.wcl	O
<b>q_level1.wcp</b>	WinCon	Run controller	q_level1.wcl	O
<b>d_level1.m</b>	MATLAB	Derive gains for conf 1	Workspace	I O
<b>q_level1.mdl</b>	SIMULINK WinCon	Controller diagram Implement controller	q_level1.wcl	O
<b>q_level1.wcp</b>	WinCon	Run controller	q_level1.wcl	O
<b>tank_cal.wcp</b>	WinCon	Calibrate for 0 level	tank_cal.wcl & mdl	I
<b>tank_cal.wcp</b>	WinCon	Calibrate for 25 cm level	tank_cal.wcl & mdl	I

## 7 System parameters

Name	Symbol	Value	Units
<b>Pump</b>			
Flow constant	Km	4.6	(cm <sup>3</sup> /sec)/Volt
Maximum Voltage	Vmax	22	Volts
Maximum flow	Fmax	100	cm <sup>3</sup> /sec
Out1 Orifice diameter	o <sub>1</sub>	.635	cm
Out2 Orifice diameter	o <sub>2</sub>	.4763	cm
<b>Tank1</b>			
Height	Lmax	30	cm
Inside Diameter		4.445	cm
Cross section area	A1	15.5179	cm <sup>2</sup>
Sensor sensitivity		5	cm/volt
<b>Tank2</b>			
Height	Lmax	30	cm
Inside Diameter		4.445	cm
Cross section area	A2	15.5179	cm <sup>2</sup>
Sensor sensitivity		5	cm/volt
<b>Outflow Orifices</b> Diameters			
Small	d <sub>1</sub>	0.3175	cm (1/8")
Medium (Typical)	d <sub>1</sub> & d <sub>2</sub>	0.47625	cm (3/16")
Large	d <sub>2</sub>	0.555625	cm (7/32")

<b>Outflow Orifices Areas</b>			
Small	$a_1$	0.0791729767	cm
Medium (Typical)	$a_1$ & $a_2$	0.17813919765	cm
Large	$a_2$	0.24246724125	cm
<b>Sensors</b>			
Pressure Range		0 - 1	PSI
Water level range		0 - 30	cm
Sensitivity	$K_s$	6.25 *	cm / Volt
Calibration nominal. You may need to recalibrate when you receive the system.			

### Configurations

Configuration	$a_1$	$L_{10}$	$a_2$	$L_{20}$	Out <sub>1</sub>	Out <sub>2</sub>	Control Output
<b>1</b>	Medium	15 cm	Medium	N/A	Tank1	N/A	$L_1$
<b>2</b>	Medium	Compute	Medium	15 cm	Tank1	N/A	$L_2$
<b>3</b>	Small	Compute	Large	10 cm	<b>Tank2</b>	<b>Tank1</b>	$L_2$