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AN ELECTRO-MECHANICAL SYSTEM OF ACTIVE SUSPENSION USING A ROBUST LQG/LTR CONTROLLER

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ABSTRACT

This paper presents an electro-mechanical system used as active suspension. A linear mathematical model for this system is presented. The aim is to improve vibration suppression in a payload carried by this suspension system. The linear quadratic gaussian / loop transfer recovery (LQG / LTR) approach is briefly explained, performance specifications are established and then the technique is applied to the linear model of the system. The target filter loop (TFL) that has the desired characteristics, as stability robustness, is obtained through the adjustment of a Kalman filter. After this, a linear quadratic regulator (LQR) is made to recover the TFL. The controlled system is evaluated with respect to vibration suppression characteristics, including situations where the payload mass value is increased or decreased in 50%. It is shown, by digital simulation, that the controlled system is robust with respect to payload mass variations.

1. Introduction

Modeling errors, parameter variations, noises and disturbances have always been the main obstacles in the design of high performance control systems. They change the behavior of the systems in unpredictable ways. In addition, on several practical applications, it is desirable that the control system improves vibration suppression with acceptable robustness to those modeling errors, parameter variations and noises. As an example of this class of applications we can mention the system to suppress mechanical vibrations.

Vibration has various detrimental effects, for that, reducing mechanical vibration provides for improved user comfort and safety, and it increases product reliability and durability by reducing wear. Nowadays, applications for which it is desired to suppress vibrations range from home appliances and automobiles to space applications and nuclear power plant (Murphy and Bailey, 1990, Campbell and Crawley, 1994, Zhou et al., 1995, Tamai and Sotelo Jr., 1995, Denoyer and Kwak, 1996, Bai and Lim, 1996, Holzhüter, 1997).

The last three decades have seen the appearance of many techniques to improve robustness, performance and stability of feedback systems. These efforts have been broadly grouped under the name of robust control. The different approaches to the synthesis of robust systems include Linear Quadratic Gaussian Loop Transfer Recovery (LQG/LTR), H2 and H optimization, Lyapunov function methods, minimax optimization and Quantitative Feedback Theory (QFT) method (Banavar and Aggarwal, 1998).

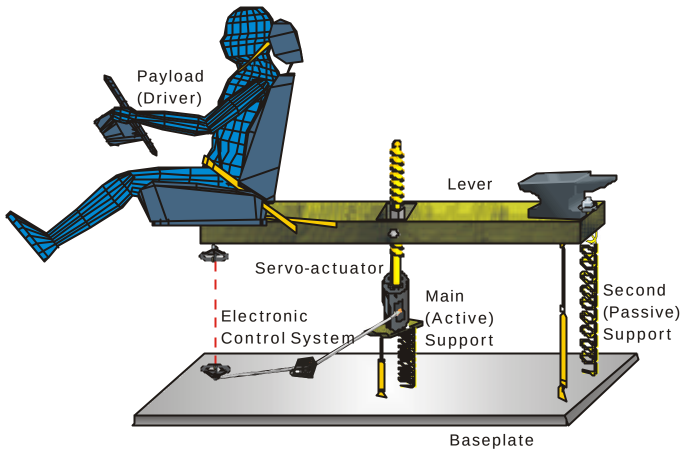

Figure 1. The proposed active suspension system.

This paper proposes an electro-mechanical system, based on the lever principle, where the aim is to provide vibration suppression of a payload (Figure 1). For that, a LQG/LTR robust controller is designed. Section 2 carries a brief and introductory historical about the LQG/LTR procedure and section 3 the electro-mechanical system model is described. Section 4 presents the performance specifications and section 5 shows the design procedure, in which the target filter loop is obtained using a Kalman filter and then it is recovered by a linear quadratic regulator adjustment. Finally, in Section 6 the implementation of controlled system is proposed and simulated in a digital computer and it is then shown that the performance and stability specifications are met.

2. An Introduction to the LQG/LTR Technique

The Linear Quadratic Gaussian approach to control design dates back to the early seventies (Athans, 1971). The robustness aspects of LQG design, and variations of this technique, were studied extensively (Gilman and Rholds, 1973; Houpis and Constantinides, 1973). Athans and Safanov (1977) showed how a multivariable LQG design can satisfy four constraints: (1) Stabilization of insufficiently stable systems; (2) Reduction of system response to noise; (3) Realization of a specific input/output relation, and; (4) Improvement of a system's robustness against variations in its open-loop dynamics. They also showed that their linear quadratic state feedback (LQSF) design had the property of an infinite gain margin and at least ±60° phase margin. The lack of guaranteed stability margins (Doyle, 1978) encouraged the effort of various researchers to develop a technique for recover the nice robustness properties of the LQSF regulator. Doyle and Stein (1979) presented a technique which they described as being an adjustment procedure for observer-based linear control system which asymptotically achieves the same loop transfer functions as full-state feedback control implementations. All these efforts resulted in the loop transfer recovery (LTR) procedure (Doyle and Stein, 1981). The basic philosophy behind recovery is that by manipulating the weighting matrices, the return ratio at the output can be made to converge to the Kalman filter return ratio or the return ratio at the input can converge to the return ratio of a Linear Quadratic Full State Feedback regulator. As a result, the nice stability properties of the Kalman filter or full state feedback regulator are obtained (Doyle and Stein, 1981, Cruz, 1996, Skogestad and Postlethwaite, 1997).

The LQG/LTR method works well mainly for minimum phase systems. To deal with nom-minimum phase systems, some modifications are essential. This is due to the fact that during recovery, the compensator obtained by the LTR procedure inverts the stable plant dynamics. In nom-minimum phase systems this would result in right half plane pole-zero cancellations which are not desirable. However, for nom-minimum phase plants this procedure can be used but with some modifications (Doyle and Stein, 1981, Athans and Stein, 1987).

3. Linear Model For The System

The proposed electro-mechanical system consists of a lever supported in two points. The main support has a DC servo-actuator to provide vertical displacements that are used for vibration suppression. The other support is passive, consisting of a spring and a damper. The lever is assumed to have a payload on the non-supported extremity. The objective is to reduce vibrations being transmitted between the baseplate and the payload. This is achieved by using the DC servo-actuator in such a way as to produce displacements that oppose the effects of the undesirable disturbances. Based in the physical assembly shown in the Figure 2 a linear model was obtained to describe the dynamical behavior of the system around an equilibrium point.

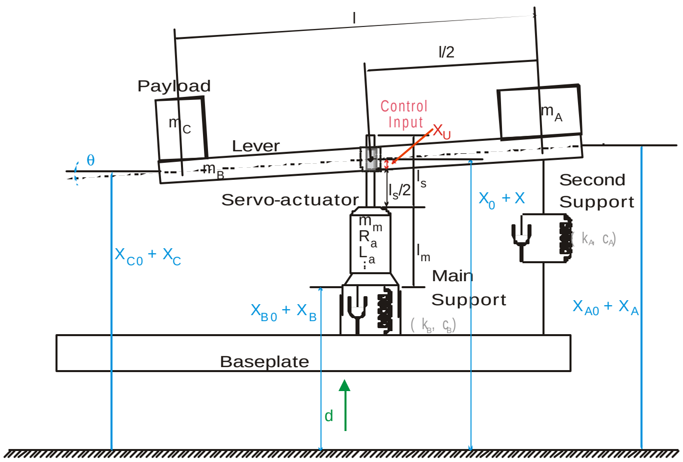

Figure 2. The physical model.

The dynamical linear model for this system may be divided in two SISO sub-models. The first, presented by equation (1) is the input/output model between a reference signal and the measured system output.

$$\begin{aligned}\dot{\mathbf{x}}_R &= \mathbf{A}_R\mathbf{x}_R + \mathbf{B}_R\mathbf{u}_R \\ Y_R &= \mathbf{C}_R\mathbf{x}_R\end{aligned} \qquad (1)$$

where:

$$\mathbf{A}_R = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & [\mathbf{M}] & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} \\ -[\mathbf{K}] & -[\mathbf{C}] & [\ddot{\mathbf{O}}_6] \\ \mathbf{0} & \mathbf{0} & [\mathbf{A}_m] \end{bmatrix} \approx$$

$$\approx \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -88{,}98 & -10{,}59 & -8{,}90 & -1{,}06 & 0{,}85 & 0{,}08 \\ -1{,}90 & -4{,}76 & -0{,}19 & -0{,}48 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -25{,}32 \end{bmatrix}$$

$$\mathbf{B}_R = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & [\mathbf{M}] & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{0} \\ [\hat{\mathbf{a}}_{1R}] \\ [\hat{\mathbf{a}}_{3R}] \end{bmatrix} \approx \begin{bmatrix} 0 & 0 & 0{,}08 & 0 & 0 & 323{,}41 \end{bmatrix}^T$$

$$\mathbf{C}_R = \begin{bmatrix} 1 & -\frac{l}{2} & \mathbf{0} \end{bmatrix} = \begin{bmatrix} 1 & -2.5 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The second sub-model, presented by equation (2) is the input/output model between a disturbance, type mechanical vibration in the baseplate, and the measured system output.

$$\begin{aligned}\dot{\mathbf{x}}_D &= \mathbf{A}_D\mathbf{x}_D + \mathbf{B}_D\mathbf{u}_D \\ Y_D &= \mathbf{C}_D\mathbf{x}_D\end{aligned} \qquad (2)$$

where:

$$\mathbf{A}_D = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & [\mathbf{M}] & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & 1 \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} \\ -[\mathbf{K}] & -[\mathbf{C}] & [\ddot{\mathbf{O}}_5] \\ \mathbf{0} & \mathbf{0} & 0 \end{bmatrix} \approx$$

$$\approx \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -88{,}98 & -10{,}59 & -8{,}90 & -1{,}06 & 88.98 \\ -1{,}90 & -4{,}76 & -0{,}19 & -0{,}48 & 1.90 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{B}_D = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & [\mathbf{M}] & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & 1 \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{0} \\ [\hat{\mathbf{a}}_{1D}] \\ [\hat{a}_{2D}] \end{bmatrix} \approx \begin{bmatrix} 0 & 0 & 8{,}90 & 0{,}19 & 1 \end{bmatrix}^T$$

$$\mathbf{C}_D = \begin{bmatrix} 1 & -\frac{l}{2} & \mathbf{0} \end{bmatrix} = \begin{bmatrix} 1 & -2{,}5 & 0 & 0 & 0 \end{bmatrix}$$

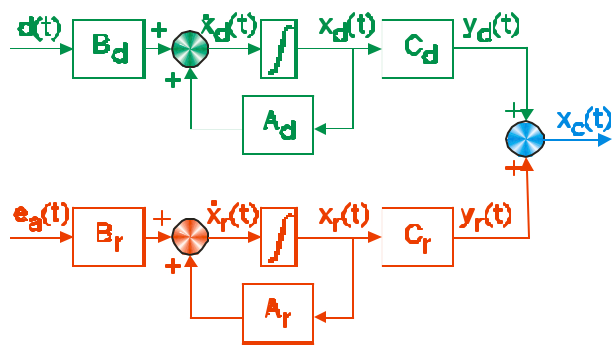

Figure 3. State space model for the proposed electro-mechanical system.

In presence of the reference and disturbance signals, the main system output, here denoted by X_C, is the sum of Y_R and Y_D, as one can see in the Figure 3.

4. Performance Specifications

The aim is to reject disturbances such as mechanical vibration. Thus the performance specifications for the controlled system are the following.

- For a disturbance of step type (maximum step size of 0.02m) the response of the controlled system should not deviate from the reference level (zero) more than 40% of the value of the injected disturbance and it should be accommodated in not more than 1 second, inside of a strip of ±5,0% of the value of the disturbance injected around this reference level;

- The controlled system should present robustness for variation up to 50% in the mass that we have desired to isolate;

- The control signal is required to be smooth and below the maximum level of 12 volts in absolute value;

5. Control Design Procedure

The LQG/LTR procedure consists of two steps: first designing the Kalman Filter in such a manner that the filter loop satisfies the performance stability robustness requirements, and second recovering this loop asymptotically by turning a full-state feedback regulator.

5.1. Target filter loop (TFL) design

The TFL design problem is to obtain the Kalman filter gain matrix, $\mathbf{K_f}$, to meet the objective of stability robustness. First, the following system is considered:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{W}x$$
$$Y = \mathbf{C}\mathbf{x} + n$$

where x, n are standard Gaussian, zero mean, white noise processes, and:

$$E\left[xx^T\right] = \hat{\mathbf{I}} = \hat{\mathbf{I}}^T \geq 0 \quad \text{and} \quad E\left[nn^T\right] = \grave{\mathbf{E}} = \grave{\mathbf{E}}^T > 0$$

The state estimates, for use in the state feedback, are obtained by a Kalman filter which is given by:

$$\left.\begin{aligned} \dot{\tilde{\mathbf{x}}} &= \mathbf{A}\tilde{\mathbf{x}} + \mathbf{B}\mathbf{u} + \mathbf{K_f}\left(\mathbf{y} - \tilde{\mathbf{y}}\right) \\ \tilde{\mathbf{y}} &= \mathbf{C}\tilde{\mathbf{x}} \end{aligned}\right\} \Rightarrow \dot{\tilde{\mathbf{x}}} = \left(\mathbf{A} - \mathbf{K_f}\mathbf{C}\right)\tilde{\mathbf{x}} + \mathbf{B}\mathbf{u} + \mathbf{K_f}\mathbf{y}$$

where the Kalman filter gain $\mathbf{K_f}$, is given by:

$$\mathbf{K_f} = \Sigma\mathbf{C}^T\Theta^{-1} \tag{3}$$

and $\mathbf{S}$ satisfies the algebraic Riccati equation:

$$\mathbf{A}\acute{\mathbf{O}} + \acute{\mathbf{O}}\mathbf{A}^T + \mathbf{W}\hat{\mathbf{I}}\,\mathbf{W}^T - \acute{\mathbf{O}}\mathbf{C}^T\grave{\mathbf{E}}^{-1}\mathbf{C}\acute{\mathbf{O}} = 0 \tag{4}$$

Using:

$$\mathbf{A} = \mathbf{A}_R{}^T;\ \mathbf{C} = \mathbf{C}_R^T;\ \mathbf{W} = \mathbf{I};\ \hat{\mathbf{I}} = \mathbf{B}_R\mathbf{B}_R^T;\ \text{and}\ \grave{\mathbf{E}} = 10^{-5}\mathbf{I}$$

where I is an identity matrix with appropriate dimension in each case, to solving the equation (3) and subsequently the equation (4), we had obtained the Kalman filter gain to the state estimation of the proposed system, and consequently the target filter loop.

5.2. Loop transfer recovery (LTR) design

After the Kalman filter gain matrix, Kf, has been obtained, the control gain matrix, KC, is calculated through the LTR. This is to calculate the full-state feedback regulator gain matrix, KC, via the optimal control technique of the control linear quadratic regulator (LQR) problem. The optimal performance measure is given as:

$$J = \int_0^\infty \left(\mathbf{x}^T\mathbf{Q}\mathbf{x} + \mathbf{u}^T\mathbf{R}\mathbf{u}\right)dt$$

where $\mathbf{Q}$ ($\mathbf{Q}$=$\mathbf{Q}^T \geq 0$) and $\mathbf{R}$ ($\mathbf{R}$=$\mathbf{R}^T > 0$) are a state weight and control weight matrix respectively. The optimal control input is given by:

$$\mathbf{u} = -\mathbf{K_C}\tilde{\mathbf{x}}$$

where the full-state feedback gain matrix, KC, is given by:

$$\mathbf{K_C} = r^{-1}\mathbf{B_R^T}\mathbf{S}$$

and $\mathbf{S}$ satisfies the following algebraic Riccati equation:

$$\mathbf{S}\mathbf{A} + \mathbf{A}^T\mathbf{S} + \mathbf{Q} - \mathbf{S}\mathbf{B}r^{-1}\mathbf{B}^T\mathbf{S} = 0 \tag{5}$$

The value of $\mathbf{K_C}$ is interactively determined by adjusting the design parameter r to recover the target filter loop. The parameters used to solve the equation (5) were:

$$\mathbf{A} = \mathbf{A}_R\ ;\ \mathbf{B} = \mathbf{B}_R;\ \mathbf{Q} = \mathbf{C}_R^{\mathrm{T}}\mathbf{C}_R;\ \mathbf{R} = \tilde{n}\mathbf{I};\ \text{with}\ r = 10^{-9}$$

where $\mathbf{I}$ is a identity matrix with appropriated dimension.

The transfer function to robust LQG/LTR controller is then given by:

$$GK(s) = \mathbf{K_C}\left(s\mathbf{I} - \mathbf{A_R} - \mathbf{B_R}\mathbf{K_C} - \mathbf{K_f}\mathbf{C_R}\right)^{-1}\mathbf{K_f}$$

and the overall closed loop systems' stable space equation can be written as:

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\tilde{\mathbf{x}}} \end{bmatrix} = \begin{bmatrix} \mathbf{A_R} & -\mathbf{B_R}\mathbf{K_C} \\ \mathbf{K_f}\mathbf{C_R} & \mathbf{A_R} - \mathbf{B_R}\mathbf{K_C} - \mathbf{K_f}\mathbf{C_R} \end{bmatrix}\begin{bmatrix} \mathbf{x} \\ \tilde{\mathbf{x}} \end{bmatrix} + \begin{bmatrix} \mathbf{W}x \\ \mathbf{K_f}n \end{bmatrix}$$

Both the optimal state feedback regulator (LQR) and the Kaman filter exhibit nice properties of infinite gain margin, least ± 60° phase margin an ½ gain reduction margin both for SISO (Athans and Safonov, 1977) as well as MIMO systems (Safonov et al., 1981). It might be expected that LQG compensator would also generally yield good robustness and performance. Unfortunately this is not so. The counterexample by Doyle (1978) proves the lack of guaranteed robustness.

Fortunately there is a way out this problem. By following the procedure LTR (Athans and Stein, 1987) these properties can be recovered. It can be shown (Doyle and Stein, 1981), by manipulating the weighting matrices, that the return ratio at the output can be made converge to the Kalman filter return ratio.

$$\lim_{r \to 0} G_R(s) GK(s) = \mathbf{C_R}\left(s\mathbf{I} - \mathbf{A_R}\right)^{-1}\mathbf{K_f}$$

where $\mathbf{C_R}\left(s\mathbf{I} - \mathbf{A_R}\right)^{-1}\mathbf{K_f}$ is called by Target Filter Loop.

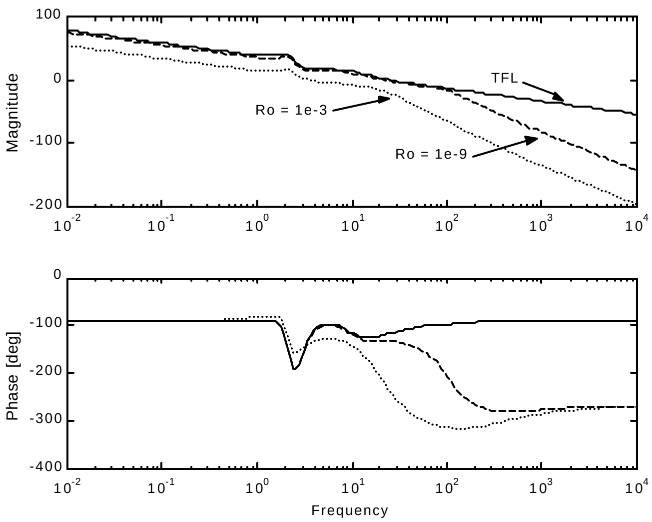

Figure 4. Convergence of LTR Procedure [$r \to 0$].

During the LTR process, while r comes close to zero the recovered loop closes to TFL, and as consequence the control inputs, u, increase so much. It is important to choose the r value that makes guaranteed a trade-off between recovery and the control signal. In this hand, the chosen value to r was 10^{-9}, as it was aforementioned.

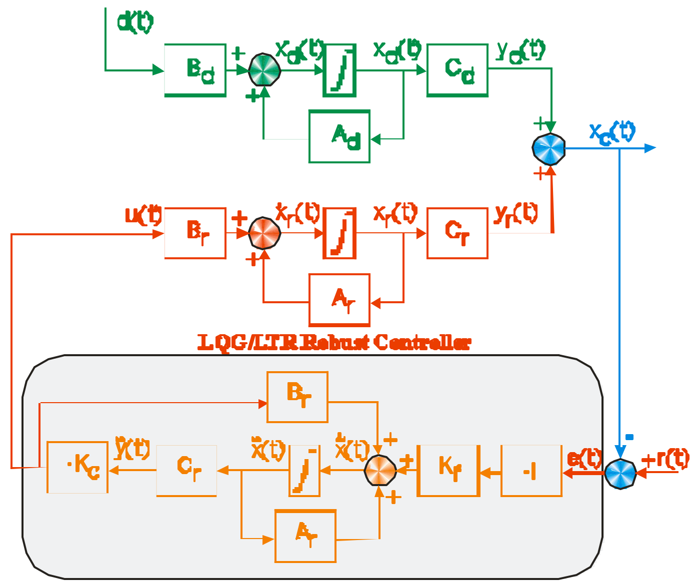

Figure 5. Controlled Closed Loop System.

6. Results

The complete system, including the LQG/LTR controller, can be represented by the block diagram shown in Figure 5.

In order to check if the controlled system satisfies the performance specification, extensive digital simulations were carried out. A disturbance signal type step of 0.01m was injected into the system with the reference fixed at zero level of displacement. Variations of $\pm 50\%$ in the mass of payload was also introduced. The results are presented in Figure 6 (system response) and Figure 7 (control signal).

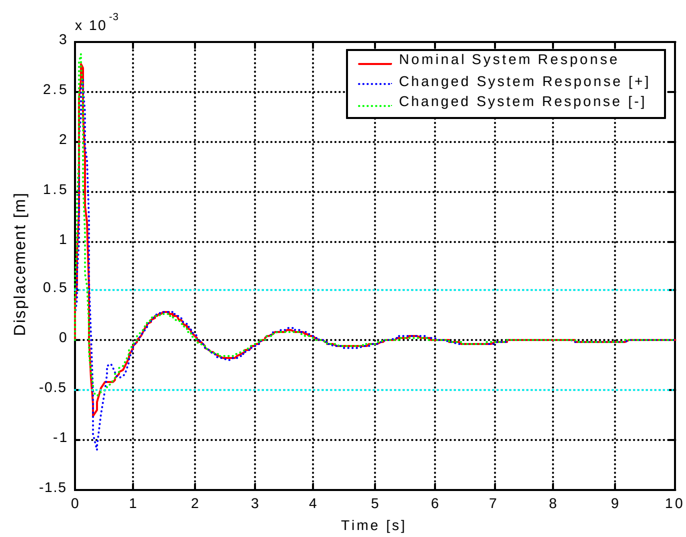

Figure 6. Response of Controlled System for a Step Disturbance, With and Without Parameter Variation.

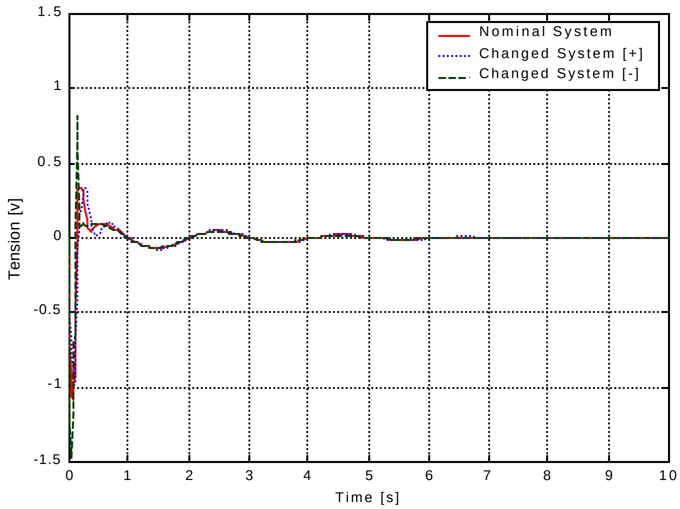

Figure 7. Input for a Step Disturbance, With and Without Parameter Variation.

One can see in Figure 6, insofar that varying the payload mass in the ± 50% the control system continues to comply with the specifications, although there was a slight deterioration of the performance, when the mass was made bigger rather then smaller. This shows the robustness of the control design.

The control input necessary to reject step disturbances is shown in Figure 7. The variation is brief and smooth, satisfying the performance specifications. As the system is linear, it is obvious that for step larger than 0,01m the amplitude of the control signal will be increased in a linear way.

Conclusions

The linear model of the proposed electro-mechanical system has been presented in state space domain. For this model, a robust controller was designed by LQG/LTR approach. Both the adjustment of the Kalman filter (which yielded the target filter loop TFL), and the selection of weighting matrices for the linear quadratic regulator (which recovered the TFL) were based on digital simulation. This was done in such a way to guarantee all the performance specifications, including the trade-off between the closed-loop response robustness and a smooth control input. The obtained controller satisfies the specification and presents the desired vibration suppression properties. Moreover, it is robust against variations in the payload mass up to 50%.

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