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MODELING, ANALYZING AND CONTROLLING OF A NONLINEAR ELECTRO-MECHANICAL SYSTEM FOR INTELLIGENT CONTROL OF VIBRATION

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Abstract. *In this paper a nonlinear electro-mechanical system, which is based on the lever principle, is modeled by Lagrange's method. The propose of this system is to improve vibration suppression between a payload and a base-plate. The nonlinear model is analyzed and a linear representation for the model is obtained around an equilibrium point. The linear model is used to investigate both, dynamics and closed loop control characteristics. A controller is designed using the linear model. Digital simulations are used to compare the nonlinear and the linear models. It is shown that, in general, controller designed using the linear model information cannot guarantee adequate performance when required to control the nonlinear system without a subsequent stage of fine tuning.*

Keywords: *Nonlinear, Modeling, Vibration, Intelligent, Controller.*

1. INTRODUCTION

The active control of vibration is an important issue in engineering. Reducing mechanical vibration may improve the user's comfort and safety, increase the product reliability and durability by reducing wear and can increase precision of pointing devices such as cameras in mobile robots. Nowadays, applications of active control of vibrations range from home appliances and automobiles to spacecrafts and nuclear power plant (Campbell and Crawley, 1994, Zhou *et al*., 1995, Denoyer and Kwak, 1996, Bai and Lim, 1996).

Several techniques have been used to control vibration. These techniques can be classified in two categories: passive and active. Active vibration control techniques serve as promising alternatives to conventional passive methods (Soong, 1990). The choice of the approach to be used in active control of vibration, basically, depends of the characteristics of the system to be controlled, of performance desired and of available tools.

Moreover, many control techniques were developed with base in linear analysis tools, and work properly with linear systems. The difficult obtaining models of nonlinear systems and working with them cannot be overlooked. A very popular way is to work with a linearized model, since there are numerous methods of analysis and several approaches to control design. Model building is the design phase where it is usually need to spend much of the time. The model is required to represent the system in a parsimonious manner. A model can be described as a useful representation of the characteristics of a plant. A model that represents with a high level of detail the desired characteristics of a plant can be considered as a model with good accuracy. However, many difficulties are intrinsic to the modeling task; for example, a model with a high level of detail, which presents a good accuracy, may not be a valuable representation of the system. Models that consider some simplifications, such as representing springs as a linear element and dampers as ideal

Coulomb frictional dissipator, may be less complex and even so they can be a good description of the system. Artificial intelligence (AI) approaches, such as fuzzy logic and neural networks, have been presented as an alternative form to work with this kind of problem (Sandri, 1999; Driankov *et.al.*, 1993; Lee, 1990; Castro, 1995; Guerra *et.al.*, 1997; Chiu and Chand, 1994; Karr and Gentry, 1993).

This paper has two main objectives. The first one is to construct a detailed nonlinear model that represents the proposed physical system and then, obtain a linear model by using Taylor's series expansion and this is shown in section 2. In section 3, the linear model is used to analyze the dynamical behavior and the control characteristics of the system. The second important objective of this work is to analyze the application of different techniques of control design applied in the linear model to obtain a controller that will be used to control the nonlinear system. Section 4 shows the results an artificial intelligence based fuzzy logic controller which was based in the design presented by Araújo et.al., 2001. These controllers are used also with the linear model as well as with the nonlinear model and the results are compared and analyzed in the section 5. Lastly, in the section 6 the conclusions of this part of the research are presented and two ways to continue this work are suggested.

2. THE ELECTRO-MECHANICAL SYSTEM

The electro-mechanical system (Araújo and Yoneyama, 2001a; Araújo and Yoneyama 2001b, Araújo and Yoneyama, 2001c) consists of a lever supported in two points. The main support has a DC servo-actuator, a spring and a damper and the second support is passive, consisting just of a spring and a damper. The lever is assumed to have a payload on the non-supported extremity and a contra-balance mass in the other extremity (see figure 1).

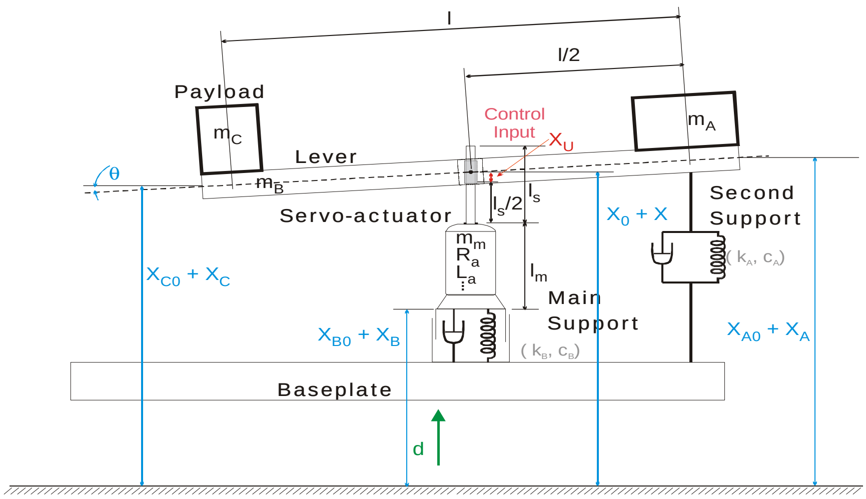


Figure 1. Physical model of the electro-mechanical system.

where: l = 5m, m_A = 100 kg, m_B = 30 kg, m_C = 100 kg, m_m = 6 kg, k_A = 1x103 N/m, c_A = 1x102 N.s/m, k_B = 2x104 N/m, c_B = 2x103 N.s/m, l_m = 0,2m, l_s = 1m, L_a = 1,22 mH e R_a = 0,13 Ω.

The objective is to reduce the transmission of vibrations between the base-plate and the payload. This is achieved by using the DC servo-actuator in such a way as to produce displacements that oppose the effects of the undesirable disturbances.

2.1. Nonlinear System Modeling

A central idea involved in the study of dynamics of real systems is the idea of a model of the system, which is simplified, abstracted constructs used to predict and analyze the system's behavior. There are many types of models, and many approaches can be used in order to obtain each type of model. In this paper a mathematical model is obtained by Lagrange's equation approach (Karnopp *et.al.*, 2000; Wellstead, 1973; Thomsom and Dahleh, 1998), where the generalized coordinates chosen were; $q_1 = x$ and $q_2 = q$.

The Lagrange's equation is given by:

$$\frac{d}{dt}\left(\frac{\partial L_{LS}(q,\dot{q})}{\partial \dot{q}_i}\right) - \frac{\partial L_{LS}(q,\dot{q})}{\partial q_i} + \frac{\partial J_{LS}(q,\dot{q})}{\partial \dot{q}_i} = F_i \tag{1}$$

Since the effects of the displacements x_u and d are already included into the Lagrangian and co-content, the terms F_i represent only the gravity's effects in the system and they are given by:

$$F_1 = k_A d_A + k_B d_B - mg \text{ ; and } F_2 = l/2(m_A - m_C)g \tag{2}$$

where, d_A and d_B are the static deformation of the springs in the supports, $m = m_A + m_B + m_C + m_m$ is the total mass of the system and g is the gravity's acceleration. Then, it is possible to represent the system by the two following differential equations:

$$\ddot{q}_1 = \frac{-k_2(T_{11} + T_{12}) + k_1 \cos(q_2)(T_{21} + T_{22})}{T_D} \tag{3}$$

$$\ddot{q}_2 = \frac{-m(T_{21} + T_{22}) + k_1 \cos(q_2)(T_{11} + T_{12})}{T_D} \tag{4}$$

where:

$$k_1 = \frac{l}{2}(m_A - m_C) \tag{5}$$

$$k_2 = \left(\frac{l}{2}\right)^2 (m_A + m_C) + \frac{1}{12} m_B (a^2 + l^2) \tag{6}$$

$$T_{11} = (k_A d_A + k_B d_B - mg) - \frac{1}{8} k_A (8q_1 + 4l \,\mathrm{sen}(q_2) - 8d) - l k_B (q_1 - x_U - d) - c_B(\dot{q}_1 - \dot{x}_U - \dot{d}) + k_1 \dot{q}_2^2 \,\mathrm{sen}(q_2) + m_m \ddot{x}_u \tag{7}$$

$$T_{12} = -\frac{\frac{1}{16} ca(8q_1 + 4l \,\mathrm{sen}(q_2) - 8d)}{4\left(q_1 + \frac{l}{2}\mathrm{sen}(q_2) - d\right)^2 + l^2(1 - \cos(q_2))^2} \left[8\left(\dot{q}_1 + \frac{l}{2}\dot{q}_2 \cos(q_2) - \dot{d}\right)\left(q_1 + \frac{l}{2}\mathrm{sen}(q_2) - d\right) + 2l^2 \dot{q}_2 \,\mathrm{sen}(q_2)(1 - \cos(q_2))\right] \tag{8}$$

$$T_{21} = \frac{l}{2}(m_A - m_C)g - \frac{1}{8}k_A\left[4\left(q_1 + \frac{l}{2}\mathrm{sen}(q_2) - d\right) l \cos(q_2) + 2l^2(1 - \cos(q_2))\mathrm{sen}(q_2)\right] \tag{9}$$

$$T_{22} = -\frac{\frac{1}{16} c_A \left[4\left(q_1 + \frac{l}{2}\mathrm{sen}(q_2) - d\right) l \cos(q_2) + 2l^2(1 - \cos(q_2))\mathrm{sen}(q_2)\right]}{4\left(q_1 + \frac{l}{2}\mathrm{sen}(q_2) - d\right)^2 + l^2(1 - \cos(q_2))^2} \left[8\left(\dot{q}_1 + \frac{l}{2}\dot{q}_2 \cos(q_2) - \dot{d}\right)\left(q_1 + \frac{l}{2}\mathrm{sen}(q_2) - d\right) + 2l^2 \dot{q}_2 \,\mathrm{sen}(q_2)(1 - \cos(q_{2)})\right] \tag{10}$$

$$T_D = -k_2 m + (k_1 \cos(q_2))^2 \tag{11}$$

Finally, the nonlinear model of the system can be written in the following form:

$$\begin{aligned} \dot{x} &= f\left(x(t),u(t),t\right) \\ y &= g\left(x(t),t\right) = q_1 - \left(l/2\right)sen(q_2) \end{aligned} \tag{12}$$

where:

$$\begin{aligned} &x(t) = \begin{bmatrix} q_1 & \dot{q}_1 & q_2 & \dot{q}_2 & x_U & \dot{x}_U & d \end{bmatrix}^{\mathrm{T}} ; u(t) = \begin{bmatrix} \ddot{x}_u & \dot{d} \end{bmatrix}^{\mathrm{T}} ; \\ &f\left(x(t),u(t),t\right) = \begin{bmatrix} \dot{q}_1 & \dfrac{-k_2\left(T_{11}+T_{12}\right)+k_1\cos(q_2)\left(T_{21}+T_{22}\right)}{T_D} & \dot{q}_2 & \dfrac{-m\left(T_{21}+T_{22}\right)+k_1\cos(q_2)\left(T_{11}+T_{12}\right)}{T_D} & \dot{x}_U & \ddot{x}_U & \dot{d} \end{bmatrix}^{\mathrm{T}} \end{aligned} \tag{13}$$

2.2. Linearization

In order to obtain a linear model for this system the eq. (12) was expanded in a Taylor's series and cutting off the higher order terms in the resulting series, the following linear model was obtained:

$$\begin{aligned} \dot{z}(t) &= \mathbf{A}z(t) + \mathbf{B}n(t) \\ y(t) &= \mathbf{C}z(t) \end{aligned} \tag{14}$$

where:

$$\mathbf{A} = \begin{bmatrix} \dfrac{\partial f_1}{\partial x_1} & \cdots & \dfrac{\partial f_1}{\partial x_7} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial f_7}{\partial x_1} & \cdots & \dfrac{\partial f_7}{\partial x_7} \end{bmatrix}_{\begin{bmatrix} x=x_{eq} \\ u=u_{eq} \end{bmatrix}} ; \mathbf{B} = \begin{bmatrix} \dfrac{\partial f_1}{\partial u_1} & \dfrac{\partial f_1}{\partial u_2} \\ \vdots & \vdots \\ \dfrac{\partial f_7}{\partial u_1} & \dfrac{\partial f_7}{\partial u_2} \end{bmatrix}_{\begin{bmatrix} x=x_{eq} \\ u=u_{eq} \end{bmatrix}} ; \mathbf{C} = \begin{bmatrix} \dfrac{\partial g}{\partial x_1} \\ \vdots \\ \dfrac{\partial g}{\partial x_7} \end{bmatrix}^{\mathrm{T}}_{\begin{bmatrix} x=x_{eq} \\ u=u_{eq} \end{bmatrix}} ; \begin{cases} x_{eq} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T ; \\ u_{eq} = \begin{bmatrix} 0 & 0 \end{bmatrix}^T ; \\ x(t) = x_{eq} + z(t) \Rightarrow x(t) = z(t); \\ u(t) = u_{eq} + n(t) \Rightarrow u(t) = n(t). \end{cases} \tag{15}$$

Using the Laplace's Transform in the eq.(14), after some manipulations, a matrix of transfer functions ($G(s)$) is obtained between the system's output ($Y(s) = X_C(s)$) and the system's inputs ($X_U(s)$ and $D(s)$).

$$Y(s) = G(s)\begin{bmatrix} X_U(s) \\ D(s) \end{bmatrix} = \begin{bmatrix} G_{x_U}(s) \\ G_d(s) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} X_U(s) \\ D(s) \end{bmatrix} = \begin{bmatrix} \dfrac{2g_{13}g_{21}+2g_{12}g_{23}-lg_{23}g_{11}-lg_{22}g_{13}}{g_{11}g_{21}-g_{12}g_{22}} \\ \dfrac{2g_{14}g_{21}+2g_{12}g_{24}-lg_{24}g_{11}-lg_{22}g_{14}}{g_{11}g_{21}-g_{12}g_{22}} \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} X_U(s) \\ D(s) \end{bmatrix} \tag{16}$$

With the polynomials g_{ij} given in term of the elements of the matrices **A** and **B** in eq.(14).

$$\begin{aligned} &g_{11} = s^2 - \frac{\partial f_2}{\partial x_2}s - \frac{\partial f_2}{\partial x_1}; g_{12} = \frac{\partial f_2}{\partial x_4}s + \frac{\partial f_2}{\partial x_3}; g_{13} = \frac{\partial f_2}{\partial u_1}s^2 + \frac{\partial f_2}{\partial x_6}s - \frac{\partial f_2}{\partial x_5}; g_{14} = \frac{\partial f_2}{\partial u_2}s + \frac{\partial f_2}{\partial x_7}; \\ &g_{21} = s^2 - \frac{\partial f_4}{\partial x_4}s - \frac{\partial f_4}{\partial x_3}; g_{22} = \frac{\partial f_4}{\partial x_2}s + \frac{\partial f_2}{\partial x_3}; g_{23} = \frac{\partial f_4}{\partial u_1}s^2 + \frac{\partial f_4}{\partial x_6}s - \frac{\partial f_4}{\partial x_5}; g_{24} = \frac{\partial f_4}{\partial u_2}s + \frac{\partial f_4}{\partial x_7}; \end{aligned}$$

2.3. The Servo-Actuator

The DC servo-actuator aforementioned consists of a DC servomotor coupled with a spindle. Models for the DC servomotor can be easily found in the area's bibliography (Ogata, 1993; Kuo 1995). In this work the model used is:

$$T_m\ddot{q}_m(t)+\dot{q}_m(t)=k_m e_{a(t)} \Rightarrow G_m(s)=\frac{\Theta_m(s)}{E_a(s)}=\frac{k_m}{T_m s^2+s} \tag{17}$$

where: K_m 12,77 and T_m 0,04.

The spindle only converts the rotational motion of the DC servomotor ($q_m(t)$) to the translational motion $x_u(t)$ by a factor given by L_P = 0,01.

3. ANALIZING THE LINEAR MODEL

3.1. Dynamic Characteristics

An *N*-DOF (N-Deegre of Freedon) system has *N* natural frequencies, and for each natural frequencies there corresponds a natural state of vibration with a displacement configuration known as the normal mode. Mathematical terms related to these quantities are known as eigenvalues and eigenvectors. Normal modes of vibrations are free undamped vibrations that depend only on the mass and stiffness of the system and how they are distributed. When vibrating at one of these normal modes, all points in the system undergo simple harmonic motion that passes through their equilibrium positions simultaneously. Several authors have wrote about the importance of doing a dynamic analysis of the systems and they have proposed methods to do this analysis (Thomson and Dahleh, 1998; Timoshenko and Young, 1948; Timoshenko *et.al.* 1974; and Meirovitch, 1967, 1990). This work follows the same steps followed in Thomson and Dahleh (1998), where the undamped free vibratiom equation is given in the following form:

$$[\mathbf{M}]\ddot{\mathbf{q}}+[\mathbf{K}]\mathbf{q}=\mathbf{0} \tag{18}$$

where M and K are the matrices of mass and shiftness of the system and they are obtained from the eqs.(3) and (4).

Such harmonic motion can be described by: $[q_1 \quad q_2]^{\mathrm{T}}=[A_1 \quad A_2]^{\mathrm{T}} e^{iw_n t}$. Then, re-writing the eq.(18), we have:

$$\big[[\mathbf{M}]w_n^2+[\mathbf{K}]\big]\ [A_1 \quad A_2]^{\mathrm{T}}=[\mathbf{Z}(l)]\ [A_1 \quad A_2]^{\mathrm{T}}=\mathbf{0}; \qquad l=w_n^2 \tag{19}$$

From eq.(19), two expression for the ratio of amplitudes are found, and by substituting the natural frequencies in either of these equations leads to the ratio of amplitudes. It is important to be noticed that this ratio equations enables us to find only the ratio of the amplitudes and not their absolute values, which are arbitrary. If one of the amplitudes is chosen equal to 1, one can say that the ratio is normalized to 1. The normalized amplitude ratio is then called the normal mode. The natural frequencies and their respective normal modes evaluated to this system were:

$$w_{n1}^2 \cong 2{,}17\,;\, w_{n2}^2 \cong 20{,}69;\ \text{and} \left(\frac{A_1}{A_2}\right)_{w_{n1,2}} = \left(\begin{bmatrix}1\\-222{,}21\end{bmatrix}_{w_{n1}^2\cong 2,17}\ \begin{bmatrix}1\\0{,}02\end{bmatrix}_{w_{n2}^2\cong 20,69}\right)$$

3.2. Control Characteristics

The analysis of characteristics like controllability, observability and stability is a very important task into control design. After obtaining a system model and analyze its dynamic, often the control designer need to change the dynamic behavior of this system to satisfies some specifications, which are called performance specifications, like speed, comfort, safety and reliability, for example. In order to decide the best way to satisfies these specifications the designer need to know the characteristics of controllability, observabilty and stability of the system to be controlled. Also after the system have been controlled it is important to analyze the stability of the controlled system.

Theorem 3.1: For the linear time invariant (LTI) system given by eq.(14) to be completely controllable, it is necessary and sufficient that the following n x nr controllability matrix has a rank of n: $\mathbf{M} = \begin{bmatrix} \mathbf{B} & \mathbf{AB} & \ldots & \mathbf{A}^{n-1}\mathbf{B} \end{bmatrix}$. Where $z(t) = x(t)$ is the n x 1 state vector, $n(t) = u(t)$ is the r x 1 input vector, and $y(t)$ is the p x 1 output vector.

In the present system, and with the values showed after the figure 1, we have that n = 7, r = 2 e p = 1 and the rank of the controllability matrix is 7. Therefore, the system is completely controllable.

With respect to the concept of observability, essentially, a system is completely observable if every state variable of the system affects some of the outputs. In other words, it is possible to obtain information of all state variables from the measurements of the outputs and the inputs. If any one of the states cannot be observed from the measurements of the system is said to be not completely observable or simply unobservable (Kuo, 1995).

Theorem 3.2: For the LTI system given by eq.(14) to be completely observable, it is necessary and sufficient that the following n x n.p observability matrix has a rank of n: $\mathbf{V} = \begin{bmatrix} \mathbf{C} & \mathbf{CA} & \ldots & \mathbf{CA}^{n-1} \end{bmatrix}^{T}$.

The rank of the observability matrix to the system described by eq.(14) is 6. Therefore, the system is completely observable in this description. In a practical sense one can see, by a brief analyze, that the measurement of the disturbances *d* can be very important to the observability of the system. Fortunately, in the model given by eq.(14) d is taken as the seventh state, consequently measure this variable consist in have two outputs to the system. Then, the matrix **C** given in eq.(15) need to be substituted by:

$$\overline{\mathbf{C}} = \begin{bmatrix} \dfrac{\partial g}{\partial x_1} & \dfrac{\partial g}{\partial x_2} & \cdots & \dfrac{\partial g}{\partial x_7} \\ 0 & 0 & \ldots & 1 \end{bmatrix}_{[x=x_{eq};u=u_{eq}]}$$

The rank of the observability matrix to the system with this new output matrix is 7. Thus, this system is completely observable. In a practical sense this result state that a model completely observable to the proposed physical system needs to measure not only the desired output (x_C) but also the disturbance injected in the system through the base-plate.

In order to analyze the stability of the non-controlled LTI system, it can be taken the frequency domain model, given by eq.(16), in closed loop. Then, the closed-loop between $X_U(s)$ and $X_C(s)$ (with $D(s) = 0$) and between $D(s)$ and $X_C(s)$ (with $X_U(s) = 0$) are given by:

$$G_{x_u}^{MF}(s) = \frac{G_{x_U}(s)}{1 + G_{x_U}(s)} = \frac{\mathrm{Num}(G_{x_U}(s)).\mathrm{Den}(1 + G_{x_U}(s))}{\mathrm{Den}(G_{x_U}(s))\mathrm{Num}(1 + G_{x_U}(s))} \quad (20)$$

$$G_D^{MF}(s) = \frac{G_D(s)}{1 + G_D(s)\left(\dfrac{G_{x_U}(s)}{G_D(s)}\right)} = \frac{G_D(s)}{1 + G_{x_U}(s)} = \frac{\mathrm{Num}(G_D(s)).\mathrm{Den}(1 + G_{x_U}(s))}{\mathrm{Den}(G_D(s))\mathrm{Num}(1 + G_{x_U}(s))} \quad (21)$$

It is clear that if the roots of the polynomial in the denominator of each eq.(20) and (21) are all in the left half *s*-plane the system will be stable with respect to the two inputs. It correspond to the real part of these roots are negative. Using the already cited data for this system we could conclude that it is stable in both situations.

4. CONTROLLING THE SYSTEM

According to Ogata (1997), linear models obtained by linearizing nonlinear system's models are important in analysis and control design of real nonlinear system. It is possible to apply numerous linear analysis methods that will produce information on the behavior of nonlinear system. This idea has been vastly used in process of analysis and control design. The second aim of this paper is to analyze the performance of a controller that was designed for the linear model, when they are working to control the nonlinear system.

A nonlinear fuzzy controller was adjusted with base on the fuzzy system proposed by Araújo *et.al.* (2001). This fuzzy system is a Takagi-Sugeno-Kang (TSK) model that has as inputs the tracking error signal and its derivative and the output is the control signal to be injected in the servo-actuator. Araújo *et.al.* (2001) present a procedure to design fuzzy controllers type TSK whose the inputs are the tracking error and its derivative, based in the relations between this structure of fuzzy controllers and PD controllers. Based in the fact that TSK first order controllers, with *n-1* inputs, given outputs such as $[C_1 \quad C_2 \quad \dots \quad C_{n-1} \quad C_n][Input_1 \quad Input_2 \quad \dots \quad Input_{n-1} \quad 1]$, they show if the inputs are the tracking error and its derivative the constants C_1 and C_2 could be interpreted as a proportional gain and a derivative gain respectively, and C_3 will represent a offset in the generated control surface. Then, this procedure (Araújo *et.al.*, 2001) shows that a TSK fuzzy controller can be designed in order to reproduce the control action of numerous PD controllers and combinations of these action. Likewise, by adding some zero order TSK function the designer can impose saturations to the controller output in order to not saturate the actuators.

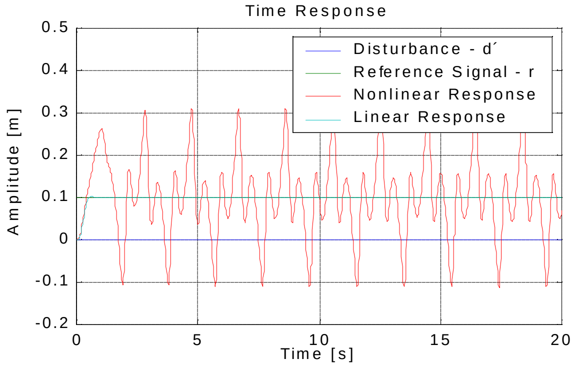
Designs of controllers are not an aim of this paper, for that, a simplified version of the TSK fuzzy controller proposed by Araújo *et.al.* (2001) is used in this work. The inputs membership functions used in the simulations here showed were the same used by Araújo *et.al.* (2001), however for the Sugeno functions on output, two different adjustments were used (Table 1). The first one (Adjustment-1) was turned only based on the linear model response and the other (Adjustment-2) was turned whit base on the nonlinear model response.

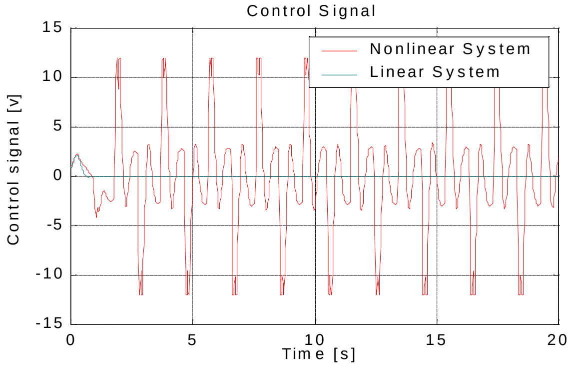
Table 1. The Sugeno output functions for the adjustment-1, with base on the linear model response, and adjustment-2 with base on the nonlinear model response.

ADJUSTMENT-1			***ADJUSTMENT-2***		
Function Name	**Function Type**	**Parameters**	**Function Name**	**Function Type**	**Parameters**
SatP	zero order	12	SatP	zero order	12
SatN	zero order	-12	SatN	zero order	-12
Linear	first order	[10 -4.9 0]	Linear	first order	[6 -2 0]

5. ANALYSIS

The systems given by eqs.(12) and (14) while using the controller with the adjustment-1, whose the parameters of the are showed in Table (1), presented the following responses:





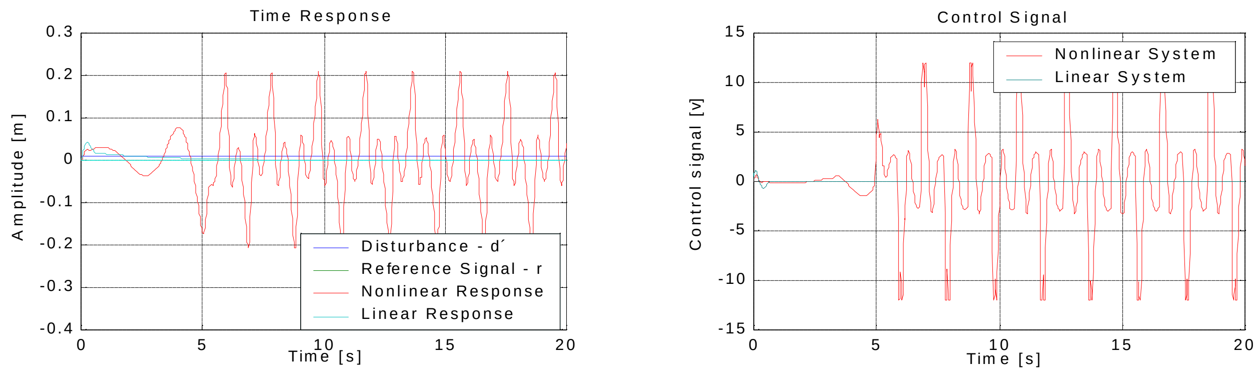


Figure 2. (a) Comparing nonlinear and linear systems response for a step reference, with the fuzzy controller. (b) Comparing nonlinear and linear systems signal control generated by the fuzzy controller to tracking a step. (c) Comparing nonlinear and linear systems response to a disturbance (*d'(t)*) type step, with the fuzzy controller. (d) Comparing nonlinear and linear systems signal control generated by the fuzzy controller to reject a step.

Now, this same systems using the controller with the adjustment-2 whose the parameters are too showed in Table (1), presented the following responses:

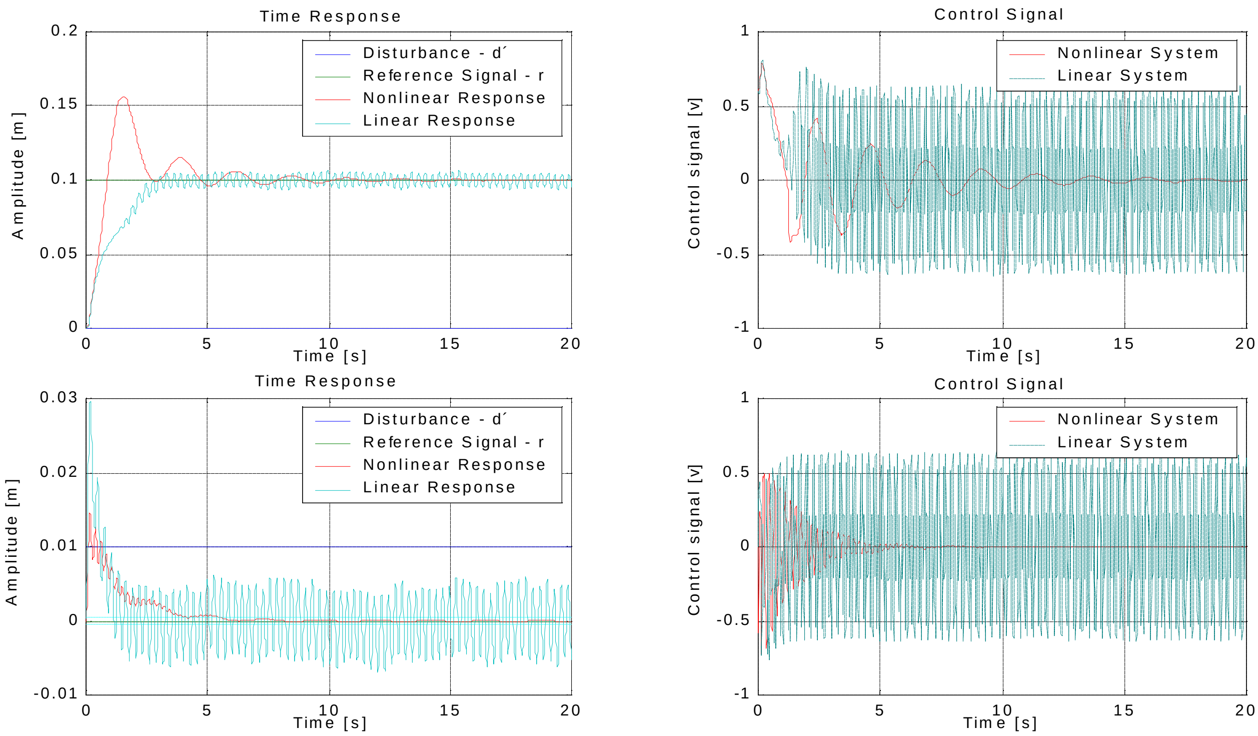


Figure 3. (a) Comparing nonlinear and linear systems response for a step reference, with the fuzzy controller. (b) Comparing nonlinear and linear systems signal control generated by the fuzzy controller to tracking a step. (c). Comparing nonlinear and linear systems response to a disturbance (*d'(t)*) type step, with the fuzzy controller (d) Comparing nonlinear and linear systems signal control generated by the fuzzy controller to reject a step.

Figures 2a-2d show that the controller turned with base on the linear model behavior does not work properly with the nonlinear system. It is probable that this occurs frequently in presence of significant nonlinearities or when the controller is not so robust with respect to the differences between the linear and the nonlinear models.

Figures 3a-3d show that it is possible to tune the controller to work properly with the nonlinear model, although one have a significant decrease on the performance of the linear model. Since the aim is to control the nonlinear system, and the linear model is used because it is simpler to manipulate, it is not important if the final controller does not work properly with the linear model. An important result shown by the fig. 3 is the fact that a controller designed with base on the linear model's information, which at first does not work properly with the nonlinear model, can be made to work properly with this nonlinear model.

6. CONCLUSIONS

As a contribution of this paper, a detailed nonlinear model was constructed for the proposed physical system. By Taylor's series expansion this model was linearized. The linear model was used in analyzing dynamics and control characteristics of the system. In this analysis the natural frequencies and the normal vibration modes were determined. It was also shown that the system is stable, controllable and observable by measuring the position of the payload ($x_C(t)$) and the position of the base ($d(t)$).

In spite of the fact that the fuzzy controller is a nonlinear system, when it was designed with base on the information of the linear model behavior it did not work properly with the nonlinear model. This fact is not surprising. Indeed, the fact of the controller can be tuned, without structural changes, to control the nonlinear system with a desired performance is another important result of this work and need to be studied with more details.

Two paths have been taken in order to continue this research. One consists in designing simple linear controllers, like PIDs, with base in the linear system and use the technique of optimal turning of controllers based on reference model, as presented in Hemerly (1996). Another way is to use an artificial neural network (ANN) to identify the unmodeled dynamics in the linear model with respect to the nonlinear model (or real plant), and this ANN would generate a compensatory control signal that would be added to the control signal generated by the controller designed with base on the linear model. The signal given by this sum could be able to control satisfactorily the nonlinear system(see Cajueiro, 2000).

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