XVI CONGRESSO BRASILEIRO DE ENGENHARIA MECÂNICA
16th BRAZILIAN CONGRESS OF MECHANICAL ENGINEERING

ACTIVE VIBRATION CONTROL BY POLE PLACEMENT: LQR X POLE DOMINANCE APPROACHES

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Abstract. *The aim of this paper is to present an active vibration control device for an electro-mechanical system consisting of a lever supported in two points. The main support, located in the middle of the lever, has a DC servo-actuator to provide vertical displacements that are used for disturbance suppression. The other support is passive, consisting of a spring and a damper. The lever is assumed to have a payload on the non-supported extremity. The objective is to reduce the transmission of vibrations between the baseplate and the payload. This is achieved by using the DC servo-actuator in such a way as to produce displacements the oppose the effects of the undesirable disturbances. Assuming low amplitude of the angular motion of the lever, a linear model of the entire electro-mechanical system is derived. The linear model is used to design a variety of controllers under a priori established performance specifications. In order to use only the payload positions as the feedback signal an observer is designed. The feedback gain matrix was determined by two different methods: Pole dominance and Linear Quadratic Regulator (LQR) approach. After each of those controllers be implemented, the robustness of complete system was checked, not only in terms of stability but also with respect to disturbance suppression. Finally, extensive computer simulations were carried out to evaluate the performance of the proposed active vibration control system.*

Keywords. *Control, Vibration, Pole Placement, Regulator, Disturbance Suppression.*

1. Introduction

The active control of vibration is an important issue in engineering. Reducing mechanical vibration may improve the user's comfort and safety, increase the product reliability and durability by reducing wear and can increase precision of pointing devices such as cameras in weapons. Nowadays, applications of actively control vibrations range from home appliances and automobiles to space applications and nuclear power plant (Murphy and Bailey, 1990, Campbell and Crawley, 1994, Zhou *et al*., 1995, Tamai and Sotelo Jr., 1995, Denoyer and Kwak, 1996, Bai and Lim, 1996, Holzhüter, 1997).

Several techniques have been used to control vibration. These techniques can be classified in two categories: passive and active. The former require the use of passive components such as vibration dampers and dynamic absorbers, which is conventional and is well developed (Rao, 1990). However, the passive approach was suffering from the major drawback of being ineffective at low frequencies. On the other hand, active control approaches provide numerous advantages; such as, better low frequency performance, smaller of size and weight, robustness to uncertainties and adaptability to unforeseen situations. Thus, active vibration control techniques serve as promising alternatives to conventional passive methods (Soong, 1990). The choice of the approach to be used in active control of vibration, basically, depends of the characteristics of the system to be controlled, of performance desired and of available tools.

This paper proposes an electro-mechanical system, based on the lever principle, where the aim is to provide vibration suppression of a payload. For that, an observer-based state feedback control is designed. Section 2 gives a brief introduction to the observer-based state feedback control approach, and Section 3 the electro-mechanical system model is described. Section 4 presents the performance specifications and the design procedure, in which the state estimator is designed by Ackermann's formula and, after that, a state feedback is designed both, the Ackermann's formula and LQR approach. In the conclusions, Section 5, the results of each one design are compared in term of the performance specifications.

2. Base-observed feedback control: A brief introduction

The developments in the mathematical theory and the evolution of the computational tools inspired many researchers, such as: Jonh C. Doyle, Günter Stein, Michael Athans, Michael G. Safonov, H. Kawakernaak, H. H. Rosembrock and many others, to dedicate their efforts to the development of new techniques to design and to analyze feedback control systems.

Safonov and Athans (1977), stated that feedback has been used in control engineering as a means for satisfying design constraints requiring:

i. Stabilization of insufficiently stable systems;
ii. Reduction of system response to noise and disturbance;
iii. Realization of a specific input/output relation (e.g., specified poles and zeros);
iv. Improvement of a system's robustness against variations in its open loop dynamics.

Safonov et. al. (1981) also noted that systems with large stability margins, good disturbance attenuation, and/or low sensitivity and described as been robust are commonly obtained using feedback. However, according to Doyle and Stein (1981), its is clear that feedback design is not trivial, because loop gains cannot be made arbitrarily high over arbitrarily large frequency ranger. Rather, they must satisfy certain performance tradeoffs and design limitations.

Feedback, in general, consists in measuring the system output and comparing it with a reference signal. This can be made so for single input/single output (SISO) systems as for multiple input/multiple output (MIMO) systems. In linear, time invariant systems, each output can be seen, mathematically, as a linear combination of some, or all, system's states. Using state feedback is possible to place closed-loop poles of linear time invariant systems.

However, the state feedback require the measurement of all states of the system that, in general, is a problem, because it is very expensive or, sometime, it is impossible to measure all states. The solution for this problem is to use a state observer to make an estimate of the states. The observer or estimator uses measurements of the control and output signals to find a good estimate to the states. The problem to do a good design of estimator is to make the error between estimated states and real states converges asymptotically for zero with enough velocity.

In fact, if a system satisfies the conditions of controllability and observability, exist many approach to design of state feedback and, consequently, of estimators. Some approaches use a desired pole structure to design, for example: Ackermann's formula, while in others approaches the pole structure is obtained by minimization of a performance index, for example the regulators.

In this paper were used the two aforementioned approaches to design so the estimator as the state feedback.

3. The linear model for the electro-mechanical system

The proposed electro-mechanical system consists of a lever supported in two points. The main support has a DC servo-actuator to provide vertical displacements that are used for vibration suppression. The second support is passive, consisting of a spring and a damper. The lever is assumed to have a payload on the non-supported extremity. The objective is to reduce the transmission of vibrations between the baseplate and the payload. This is achieved by using the DC servo-actuator in such a way as to produce displacements that oppose the effects of the undesirable disturbances (see Figure 1).

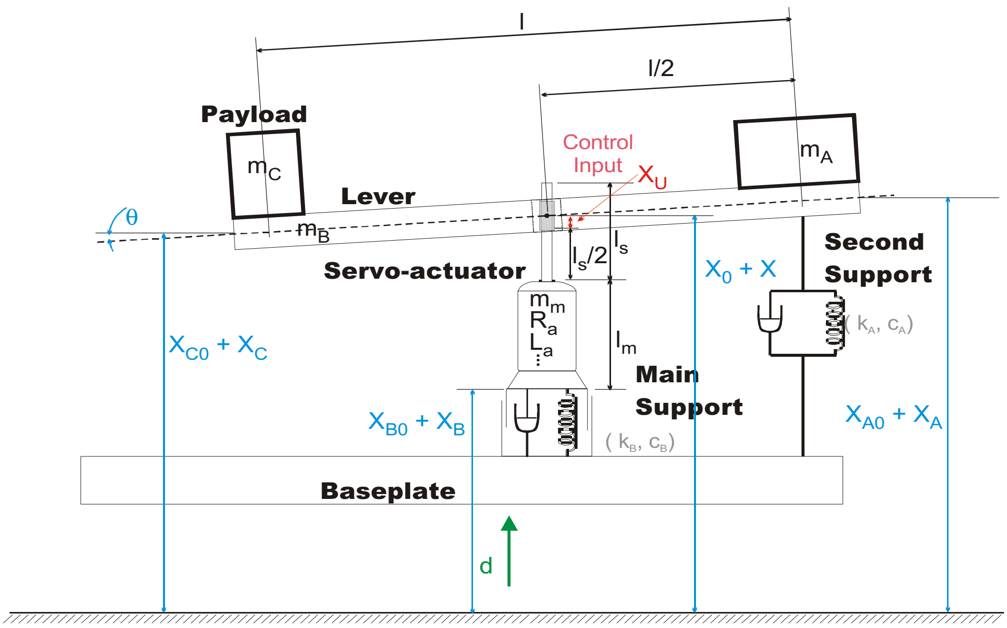

Figure 1. The proposed electro-mechanical system.

where:

$l = 5$m, $m_A = 100$ kg, $m_B = 30$ kg, $m_C = 100$ kg, $m_m = 6$ kg,
$k_A = 1\text{x}10^3$ N/m, $c_A = 1\text{x}10^2$ N.s/m, $k_B = 2\text{x}10^4$ N/m, $c_B = 2\text{x}10^3$ N.s/m,
$l_m = 0{,}2$m, $ls = 1$m, $L_a = 1{,}22$ MH e $R_a = 0{,}13\ \Omega$.

The dynamical linear model for this system may be divided in two SISO sub-models. The first, represented by equation (1) is the input/output model between a reference signal and the measured system output.

$$\begin{aligned}\dot{\mathbf{x}}_R(t) &= \mathbf{A}_R\mathbf{x}_R(t) + \mathbf{B}_R\mathbf{u}_R(t)\\ Y_R(t) &= \mathbf{C}_R\mathbf{x}_R(t)\end{aligned} \tag{1}$$

where:

$$\mathbf{A}_R = \left[\begin{array}{cc|c} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & [\mathbf{M}] & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{0} & \mathbf{I} \end{array}\right]^{-1} \left[\begin{array}{cc|c} \mathbf{0} & \mathbf{I} & \mathbf{0} \\ -[\mathbf{K}] & -[\mathbf{C}] & [\mathbf{P}_1] \\ \hline \mathbf{0} & \mathbf{0} & [\mathbf{A_m}] \end{array}\right] \approx \left[\begin{array}{cccc|cc} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -88{,}98 & -10{,}59 & -8{,}90 & -1{,}06 & 0{,}85 & 0{,}08 \\ -1{,}90 & -4{,}76 & -0{,}19 & -0{,}48 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -25{,}32 \end{array}\right]$$

$$\mathbf{B}_R = \left[\begin{array}{cc|c} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & [\mathbf{M}] & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{0} & \mathbf{I} \end{array}\right]^{-1} \left[\begin{array}{c} \mathbf{0} \\ [\mathbf{P}_2] \\ \hline [\mathbf{P}_3] \end{array}\right] \approx \left[\begin{array}{cccc|cc} 0 & 0 & 0{,}08 & 0 & 0 & 323{,}41 \end{array}\right]^T$$

$$\mathbf{C}_R = \left[1 \quad -\frac{l}{2} \quad \mathbf{0}\right] = \left[1 \quad -2.5 \quad 0 \quad 0 \quad 0 \quad 0\right]$$

The state vector $\mathbf{x_R}(\boldsymbol{t})$, which will be estimated, is given by:

$\mathbf{x}_R = \left[x \quad \theta \quad \dot{x} \quad \dot{\theta} \quad \theta_m \quad \dot{\theta}_m\right]$; where θ_m is the angular velocity of the DC motor.

The second sub-model, represented by equation (2) is the input/output model between a disturbance, type mechanical vibration in the baseplate, and the measured system output.

$$\begin{aligned} \dot{\mathbf{x}}_D(t) &= \mathbf{A}_D \mathbf{x}_D(t) + \mathbf{B}_D \mathbf{u}_D(t) \\ Y_D &= \mathbf{C}_D \mathbf{x}_D(t) \end{aligned} \tag{2}$$

where:

$$\mathbf{A}_D = \left[\begin{array}{cc|c} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & [\mathbf{M}] & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{0} & 1 \end{array}\right]^{-1} \left[\begin{array}{cc|c} \mathbf{0} & \mathbf{I} & \mathbf{0} \\ -[\mathbf{K}] & -[\mathbf{C}] & [\mathbf{P}_4] \\ \hline \mathbf{0} & \mathbf{0} & 0 \end{array}\right] \approx \left[\begin{array}{cccc|c} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -88{,}98 & -10{,}59 & -8{,}90 & -1{,}06 & 88.98 \\ -1{,}90 & -4{,}76 & -0{,}19 & -0{,}48 & 1.90 \\ \hline 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\mathbf{B}_D = \left[\begin{array}{cc|c} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & [\mathbf{M}] & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{0} & 1 \end{array}\right]^{-1} \left[\begin{array}{c} \mathbf{0} \\ [\mathbf{P}_5] \\ \hline [\mathbf{P}_6] \end{array}\right] \approx \left[\begin{array}{cccc|c} 0 & 0 & 8{,}90 & 0{,}19 & 1 \end{array}\right]^T$$

$$\mathbf{C}_D = \left[1 \quad -\frac{l}{2} \quad \mathbf{0}\right] = \left[1 \quad -2{,}5 \quad 0 \quad 0 \quad 0\right]$$

In presence of the reference and disturbance signals, the main system output, here denoted by X_C, which measured in meters, is the sum of Y_R and Y_D, as one can see in the Figure 2.

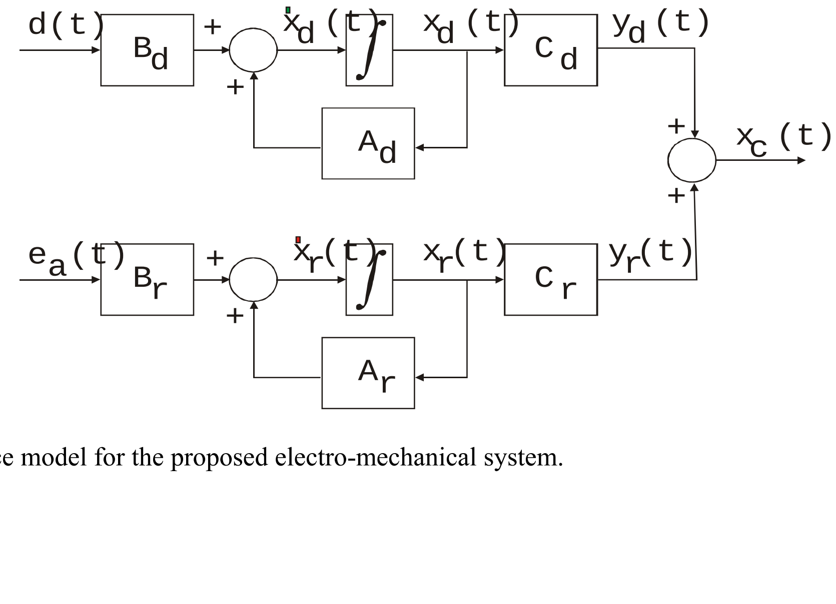

Figure 2. State space model for the proposed electro-mechanical system.

4. Control design

The control design procedure in this paper present three important phases; I – In the first phase, the desired performance in term of speed, security and reliability were expressed in term of characteristics of step response such as overshoot, settling time, rise time and steady state error. II – A state estimator was designed to provide an estimate of the states at each instant. III – finally, a state feedback gain matrix completed the state feedback control. In this phase, two different approaches were used to design; the pole dominance and the LQR approach.

4.1. Performance specifications for the controlled system

The aim here is to reject disturbances in the form of mechanical vibrations. Thus the performance specifications for the controlled system are presented in terms of vibration suppression.

- For a disturbance of step type (maximum step size of 0.05m) the response of the controlled system should not deviate from the reference level (zero) more than 40% of the value of the injected disturbance and it should be accommodated in not more than 3 second, inside of a strip of ± 5% of the value of the disturbance injected around this reference level;

- The controlled system should present robustness for variation up to 50% in the payload's mass;

- The control signal is required to be smooth and below the maximum level of 12 volts in absolute value;

4.2. States estimator design

The estimator is necessary to make possible the state feedback control. Notice, in eq. (1), that there are six states and only one output. If it were possible to measure all the states it would be necessary to have six sensors, and this would increase the implementation and operational costs.

The state estimator was designed using the dual system to that given by the eq. (1), i.e., $\mathbf{A} = \mathbf{A}_R^T$; $\mathbf{B} = \mathbf{C}_R^T$ and $\mathbf{C} = \mathbf{B}_R^T$, to find the gain matrix **K**, where the estimator gain matrix, **L**, is given by $\mathbf{L} = \mathbf{K}^T$. The matrix **L** is used in the estimator to generate a correction signal with respect to modeling uncertainty.

The aim is to locate the poles of $\tilde{\mathbf{A}}$ in a desired region, where $\tilde{\mathbf{A}} = \mathbf{A} - \mathbf{BK}$. This is possible if the system in eq. (1) is observable or, likewise, the dual system is controllable. It was verified that this condition is satisfied and the controllability matrix of the dual system, $\overline{\mathbf{M}}$, was calculated by:

$$\overline{\mathbf{M}} = \left[\mathbf{C}_R^{\ T} \quad \mathbf{A}_R^{\ T}\mathbf{C}_R^{\ T} \quad \dots \quad \left(\mathbf{A}_R^{\ T}\right)^5 \mathbf{C}_R^{\ T}\right] = \left[\mathbf{B} \quad \mathbf{AB} \quad \dots \quad \mathbf{A}^5\mathbf{B}\right] \tag{3}$$

Based on the desired pole structure, the characteristic equation was determined:

$$\det\left(s\mathbf{I} - \tilde{\mathbf{A}}\right) = \det\left(s\mathbf{I} - \mathbf{A} + \mathbf{BK}\right) = \left(s - \mathrm{m}_1\right)\left(s - \mathrm{m}_2\right)\dots\left(s - \mathrm{m}_N\right) = s^n + \mathrm{a}_1 s^{n-1} + \dots + \mathrm{a}_{n-1}s + \mathrm{a}_n = 0 \tag{4}$$

where, $\left(\mathrm{m}_1, \mathrm{m}_2, \dots, \mathrm{m}_N\right)$ are the desired poles of $\left(\mathbf{A} - \mathbf{BK}\right)$.

The Cayley-Hamilton theorem (Kirk, 1970) states that $\tilde{\mathbf{A}}$ satisfy your own characteristic equation, then:

$$\mathrm{f}\left(\tilde{\mathbf{A}}\right) = \tilde{\mathbf{A}}^n + \mathrm{a}_1\tilde{\mathbf{A}}^{n-1} + \dots + \mathrm{a}_{n-1}\tilde{\mathbf{A}} + \mathrm{a}_n\mathbf{I} = \mathbf{0} \tag{5}$$

where:

$$\tilde{\mathbf{A}}^n = \left(\mathbf{A} - \mathbf{BK}\right)^n = \mathbf{A}^n - \mathbf{A}^{n-1}\mathbf{BK}\tilde{\mathbf{A}}^{n-n} - \mathbf{A}^{n-2}\mathbf{BK}\tilde{\mathbf{A}}^{n-(n-1)} - \dots - \mathbf{A}^{n-n}\mathbf{BK}\tilde{\mathbf{A}}^{n-1} \tag{6}$$

Now, it is possible to obtain of the eqs. (5) and (6):

$$\mathrm{f}(\mathbf{A}) = \left[\mathbf{B} \;\vdots\; \mathbf{AB} \;\vdots\; \dots \;\vdots\; \mathbf{A}^{n-2}\mathbf{B} \;\vdots\; \mathbf{A}^{n-1}\mathbf{B}\right] \begin{bmatrix} \mathrm{a}_{n-1}\mathbf{K} + \mathrm{a}_{n-2}\mathbf{K}\tilde{\mathbf{A}} + \dots + \mathrm{a}_1\mathbf{K}\tilde{\mathbf{A}}^{n-2} + \mathbf{K}\tilde{\mathbf{A}}^{n-1} \\ \mathrm{a}_{n-2}\mathbf{K} + \mathrm{a}_{n-3}\mathbf{K}\tilde{\mathbf{A}} + \dots + \mathrm{a}_1\mathbf{K}\tilde{\mathbf{A}}^{n-3} + \mathbf{K}\tilde{\mathbf{A}}^{n-2} \\ \vdots \\ \mathrm{a}_1\mathbf{K} + \mathbf{K}\tilde{\mathbf{A}}^1 \\ \mathbf{K} \end{bmatrix} \tag{7}$$

As the system is controllable, the inverse of $\overline{\mathbf{M}}$ exist. Lastly, the Ackermann's formula is obtained multiplying both sides of eq. (7) by $\mathbf{K} = [0 \vdots 0 \vdots \ldots \vdots 0 \vdots 1]\mathbf{M}^{-1}\mathrm{f}(\mathbf{A})$.

$$\mathbf{K} = [0 \vdots 0 \vdots \ldots \vdots 0 \vdots 1]\mathbf{M}^{-1}\mathrm{f}(\mathbf{A}) \tag{8}$$

After several interactions in a process with the following steps:

1. Choice of a desired pole structure;
2. Design by Ackermann's formula;
3. Analysis by simulation;
4. If the result of the analyze was satisfactory then the design is finish, else choice a new pole structure and return to step 2.

The estimator gain matrix calculated was:

$$\mathbf{L} = \mathbf{K}^T \approx [178{,}15 \quad 53{,}94 \quad 85{,}27 \quad -238{,}82 \quad 21765{,}08 \quad -4455{,}59]^T \tag{9}$$

The pole structure used to design the state estimator was based in pole dominance. A pair of complexes poles have been located as closed to the imaginary axis as possible and the others four poles have been chosen to be real poles as far from the dominates poles as possible. The design aim is to obtain a fast state estimator. It is possible by using larges gains. If a large gain is used in the state estimator the feedback gain to be designed is desired to be so lower as possible.

4.3. State feedback gain matrix

After the estimator has been designed, it is possible to implement the state feedback control (Figure 3). The state feedback gain matrix K was designed by two approaches: by pole dominance and by LQR approach.

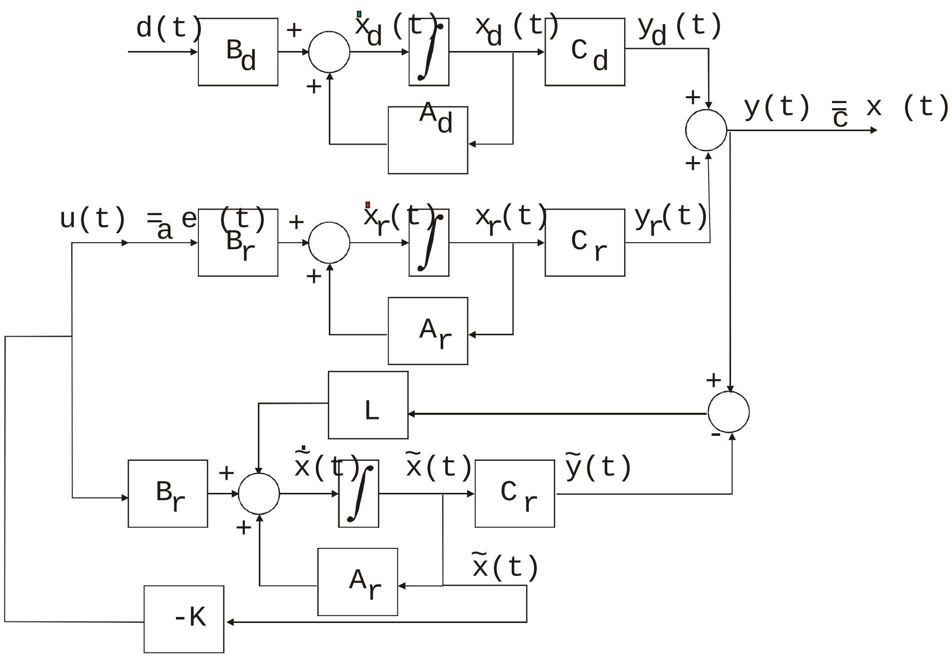

Figure 3. Base-observed feedback control for the proposed electro-mechanical system.

4.3. 1. Design by Ackermann's formula

The procedure to obtain the state feedback gain matrix, **K**, to the system in eq. (1), that satisfies the performance specifications given in section 4.1, is same that followed in the design of the estimator, but in this case we have that: $\mathbf{A} = \mathbf{A}_R$; $\mathbf{B} = \mathbf{B}_R$ and $\mathbf{C} = \mathbf{C}_R$.

As said therefore, the desired pole structure is needed to apply Akermann's formula. If a system has more than one pair of complexes dominants poles, finding a pole structure that provides a system response that satisfies the performance specifications is not trivial. For that, the class of pole structure analyzed had one dominant pair of poles, and the others poles were located on the real axis.

Based in classical relations to a second order system time response an initial estimate of desired pole structure was found. After this, by computations simulation, the initial estimate was improved and the final desired pole structure was determined as:

$$\mathrm{m}_{1,2} = -0{,}3 \pm 3j;\ \mathrm{m}_3 = -100;\ \mathrm{m}_4 = -110;\ \mathrm{m}_5 = -120;\ \mathrm{m}_6 = -130 \tag{10}$$

Using this pole structure to determine the characteristic equation $f(\tilde{\mathbf{A}})$, by Ackermann's formula the state feedback gain matrix, **K**, was obtained as:

$$\mathbf{K} \cong \begin{bmatrix} 2{,}90x10^5 & -1{,}31x10^6 & -4{,}36x10^4 & 2{,}21x10^5 & 3{,}761x10^3 & 1{,}24x10^1 \end{bmatrix} \tag{11}$$

Using the matrix **L** (eq. (9)) designed in section 4.2 and this matrix **K** (eq. (11)) in a computational simulation by MATLAB/SIMULINK using the diagram presented in Figure 3 and with the state estimator initial conditions equals to zero, it was possible to see that the controlled system satisfies the performance specifications (Figure 4).

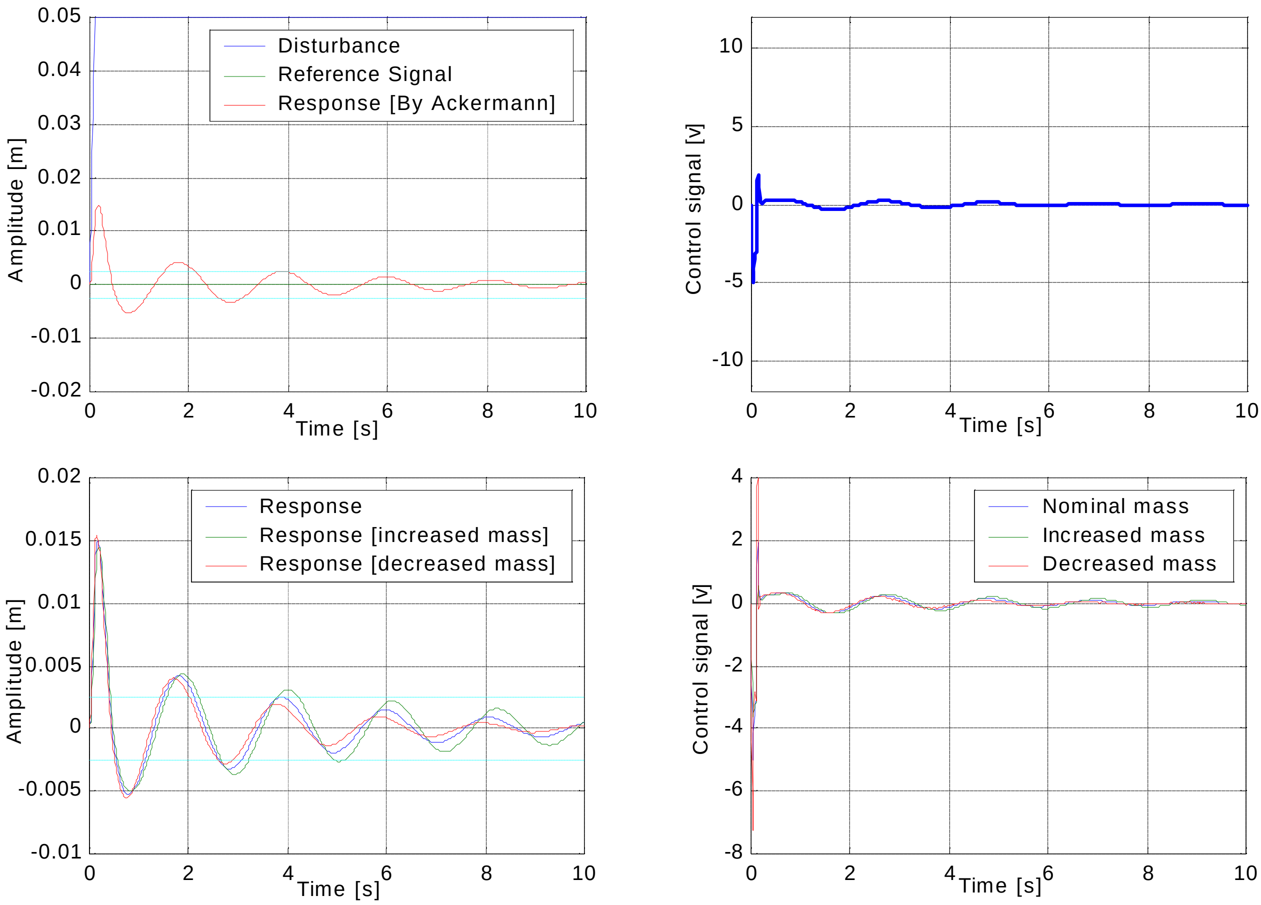

Figure 4(a). Controlled system response. State feedback design by Ackermann's formula.
Figure 4(b). Control signal. State feedback design by Ackermann's formula.
Figure 4(c). Robustness with respect to a variation of ±50% in the mass of the payload. State feedback design by Ackermann's formula.
Figure 4(d). Control signal with variation of ±50% in the mass of the payload. State feedback design by Ackermann's formula.

In the Figure 4(a) we can see that the response of the controlled system didn't deviates from the reference level more than 30% of the value of the disturbance and it was accommodated, inside of a strip of ± 5,0% of the value of the disturbance, in approximately 3 seconds. We can still see, in Figure 4(b), that this result was obtained with a control sign whose width doesn't exceed 12 volts. Lastly, with respect to the robustness of the system with respect to parametric uncertainties can be seen in Figures 4(c) and 4(d) that variations of 50% in the mass of the payload just cause small variations in the system response and in the control signal. In the system response the increase of the mass causes more deterioration than its decrease. In both cases the deterioration is not significant when the deteriorated signal is compared with the nominal signal, but in term of performance specifications, the increase of the mass increases the time needed to the response go into in the aforementioned strip of 5,0%, from 3 seconds to more than 4 seconds and, therefore, the system in this situation don't satisfies the given specifications. This occurs because the nominal system response is just closed in this specification, this could be improved, but the gains in the matrix **L** would increase. It is well known in the bibliography that when the gains tend to infinity the solution tends to ideal, but as aforesaid high gains are not desired.

4.3.2. Design by LQR approach

The quadratic optimal regulator problem consists in finding a state feedback gain matrix, **K**, by minimization of a quadratic cost functional. Thus, in this approach, it is not necessary to know the pole structure, because the minimization of the functional gives automatically this structure. Moreover, for a large class of control problems, it is possible to show a direct relation between the quadratic cost functional used in optimal design and the Liapunov's functions. In fact, an approach used to design LQRs can be based on second Liapunov method, which assures the stability of the controlled system. In general, the the quadratic cost functional is given by:

$$J = \int_0^\infty \left(\mathbf{x}^T(t)\mathbf{Q}\mathbf{x}(t) + \mathbf{u}^T(t)\mathbf{R}\mathbf{u}(t) \right) dt \tag{12}$$

where **Q** is a real symmetric positive semi-definite matrix and **R** is a real symmetric positive definite matrix which can be written by $\mathbf{R} = \mathbf{T}^T\mathbf{T}$. These matrices, in that order, correspond to the weight of the states and control signal in the cost J in each moment.

It is well-known that control signal is given by:

$$\mathbf{u}(t) = -\mathbf{K}\mathbf{x}(t) \tag{13}$$

By substituting the eq. (13) in eq. (12), are gets:

$$J = \int_0^\infty \left(\mathbf{x}^T\mathbf{Q}\mathbf{x} + \mathbf{x}^T\mathbf{K}^T\mathbf{R}\mathbf{K}\mathbf{x} \right) dt = \int_0^\infty \mathbf{x}^T\left(\mathbf{Q} + \mathbf{K}^T\mathbf{R}\mathbf{K} \right)\mathbf{x}dt = \int_0^\infty \mathbf{x}^T\left(\mathbf{Q} + \mathbf{K}^T\mathbf{T}^T\mathbf{T}\mathbf{K} \right)\mathbf{x}dt \tag{14}$$

If (**A-BK**) is stable, then the second Liapunov method assure that exist a real symmetric positive definite matrix, **P**, that can be obtained by solving the following algebraic Riccati equation (ARE):

$$\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} + \left[\mathbf{T}\mathbf{K} - \left(\mathbf{T}^T\right)^{-1}\mathbf{B}^T\mathbf{P}\right]^T\left[\mathbf{T}\mathbf{K} - \left(\mathbf{T}^T\right)^{-1}\mathbf{B}^T\mathbf{P}\right] - \mathbf{P}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P} + \mathbf{Q} = 0 \tag{15}$$

From eq. (15), we can see that the minimal value for the cost J with respect to **K** is obtained when $\mathbf{x}^T\left[\mathbf{T}\mathbf{K} - \left(\mathbf{T}^T\right)^{-1}\mathbf{B}^T\mathbf{P}\right]^T\left[\mathbf{T}\mathbf{K} - \left(\mathbf{T}^T\right)^{-1}\mathbf{B}^T\mathbf{P}\right]\mathbf{x} = 0$. As a result of this **K** is given by:

$$\mathbf{K} = \mathbf{T}^{-1}\left(\mathbf{T}^T\right)^{-1}\mathbf{B}^T\mathbf{P} = \mathbf{R}^{-1}\mathbf{B}^T\mathbf{P} \tag{16}$$

In the present design the cost functional used is given by:

$$J = \int_0^\infty \left(y^T\, y + \mathbf{u}^T\mathbf{R}\mathbf{u} \right) d = \int_0^\infty \left(\mathbf{x}^T\mathbf{C}_R{}^T\, \mathbf{C}_R\mathbf{x} + \mathbf{u}^T\mathbf{R}\mathbf{u} \right) dt = \int_0^\infty \left(\mathbf{x}^T\mathbf{C}_R{}^T\, \mathbf{C}_R\mathbf{x} + \mathbf{u}^T\,\mathrm{b}\,\mathbf{I}\mathbf{u} \right) dtt \tag{17}$$

where I is a identity matrix with appropriated dimension, and the factor b give the relation between the weight of the states and the weight of the control signal in the cost functional, and it is responsible for the adjustment of the functional to the performance specifications given.

Based in the theory here presented, we can summarize the steps of this design as presented:

1. Choice a value to the factor b;
2. Solve the ARE in equation (15) to determine the state feedback gain matrix **K** (equation (16));
3. Analysis by simulation of the controlled system;
4. If the controlled system satisfy the performance specifications then the design is finish, else choice a new value to the factor b and return to step 2.

After several simulations the chosen value for the factor b and the matrix K determined were:

$$\mathrm{b} = 10^{-8} \tag{18}$$

$$\mathbf{K} \cong \begin{bmatrix} 2{,}93x10^3 & -7{,}90x10^3 & 1{,}12x10^2 & -3{,}43x10^2 & 2{,}32 & 1{,}80x10^{-1} \end{bmatrix} \tag{19}$$

Now, using the matrix **L** (eq. (9)) designed in section 4.2 and this matrix **K** (eq.(19)) in a computational simulation, in the same conditions early presented, it was possible to obtain the results presented in Figure 5.

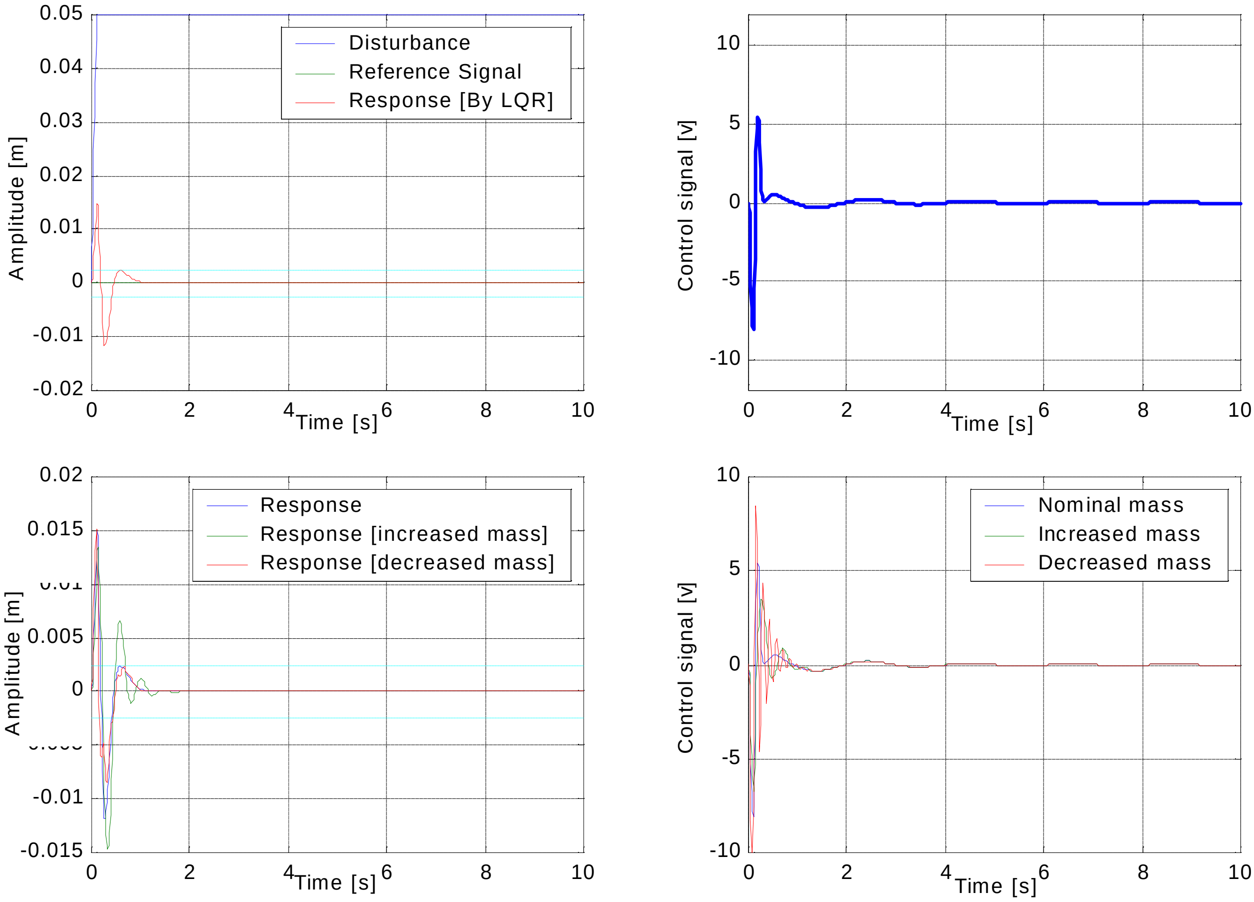

Figure 5(a). Controlled system response. State feedback design by LQR approach.

Figure 5(b). Control signal. State feedback design by LQR approach.

Figure 5(c). Robustness with respect to a variation of ±50% in the mass of the payload. State feedback design by LQR approach.

Figure 5(d). Control signal with variation of ±50% in the mass of the payload. State feedback design by LQR approach.

Now, with respect to the LQR design, in Figure 5(a) we can see that, likewise to the dominant pole formula design, the response of the controlled system didn't deviate from the reference level more than 30% of the value of the disturbance however, using the LQR designed control, it was accommodated, inside of a strip of ± 5,0% of the value of the disturbance, in just a little more than 0,4 seconds. We can still see, in Figure 5(b), that the control signal generated by the LQR controller doesn't exceed 12 volts too. The behavior of the controlled system and of the control signal, with respect to variations in the mass of the payload, in the design by LQR approach, can be seen in Figures 6(c) e 6(d), notice that the increase of the mass cause deterioration in the response of the controlled system by the augment of the time necessary to accommodate this response, from in the order of 0,4 to almost 0,7 seconds, but as this characteristic in nominal response (0,4 sec.) is very better than the expected value, for that, this deterioration is not so problematic. In this design the main problem is the decrease of the mass. Notice, in Figures 6(c) e 6(d), that when the mass is decreased a second, and faster, mode of vibration appears, it can be an undesired characteristic in many applications, moreover, to try control this fast mode demands a oscillatory control signal (Figure 5(d)) which in practical applications can be impossible to be obtained.

It is obvious, by studying the references in this topic, that decreasing the factor b in the design by LQR approach, the resultant control will be more effective than this, but it implicates in larger gains in matrix **K** and, as a result, lager amplitudes in the control signal, Moreover, a increment of the robustness isn't assured.

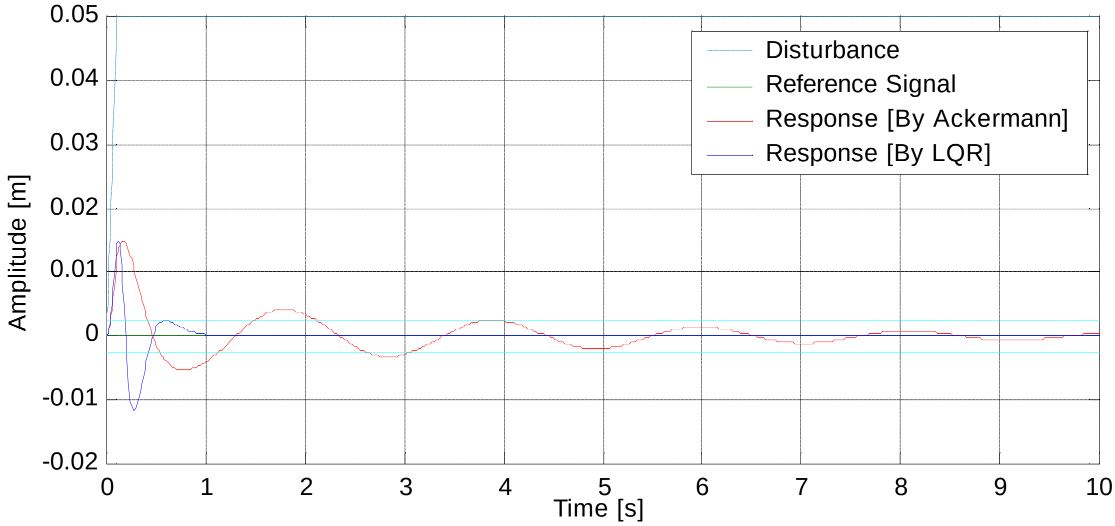

Figure 6. Controlled system response. The LQR approach versus Ackermann's formula.

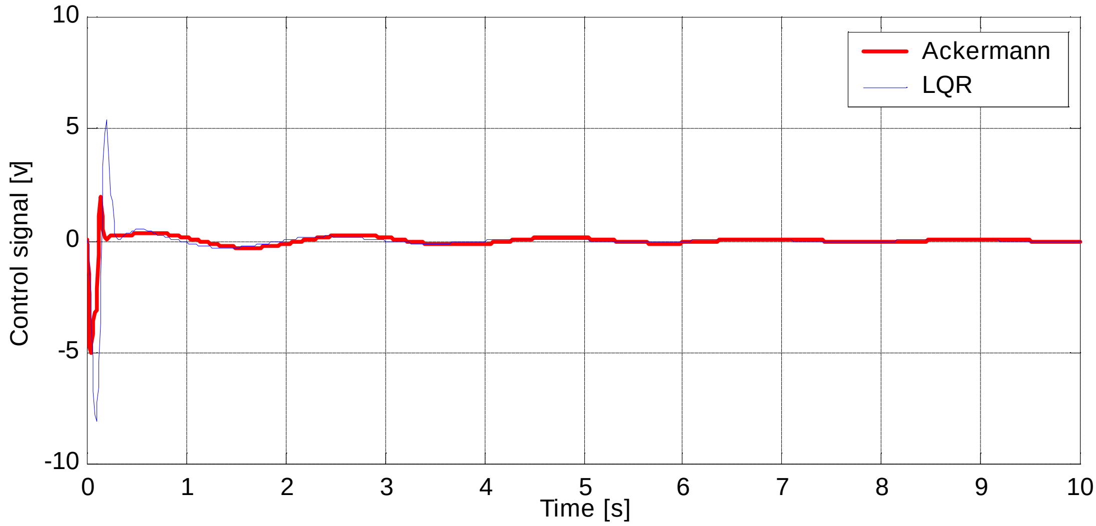

Figure 7. Control signal. The LQR approach versus Ackermann's formula.

5. Conclusions

We can see, by comparing the controlled system response (Figure 6) and the control signal (Figure 7) generated in both design, that the LQR controlled system present a faster decrement than the other design, where decrement is the ratio of any two successive amplitudes. On the other hand, this characteristic is obtained by larger amplitude of control system than in Ackermann's formula design. As the control signal generated by LQR design doesn't exceed the limit of 12 volts to the amplitude of disturbance specified, this is a not major problem.

Both the designed controls present a good behavior in nominal conditions. In these conditions, both satisfied the performance specifications, but the LQR approach yielded smaller gains in the state feedback gain matrix and a better performance than the pole dominance design. However, this second generates a smaller control signal.

When the mass of payload present variations up to 50%, it was verified that the robust stability is satisfactory in both designs, but the robust performance is not obtained in the pole dominance design. Another problem is this approach is the difficulty to find a desired pole structure. The effects of the variations in this structure, is far from clear, in the state feedback gain matrix and, consequentially, in the system response. For that, the LQR design is considered here, better than the Ackermann's formula design.

In the next step of this research it is expected to carry out a linked design between estimator and state feedback using, for example, the linear quadratic gaussian with loop transfer recovery (LQG/LTR). With this approach it is expected to obtain more robustness.

6. Acknowledgment

The authors are grateful to FAPESP (Fundação de Amparo à Pesquisa do Estado de São Paulo) for the financial support rendered trough the process 99/02409-4.

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